

M242 Q11(b) (S. Zhang) (8 points). Name: \_\_\_\_\_

1. Find the limit by both methods: (1) L'Hopital rule, (2) Taylor series expansion.

$$\lim_{x \rightarrow 0} \frac{\sin x - x - x^2}{\cos x - 1}$$

• ans: (1) 0/0.

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin x - x - x^2}{\cos x - 1} &\stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{\cos x - 1 - 2x}{-\sin x} \\ &\stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{-\sin x - 2}{-\cos x} \\ &= \frac{-2}{-1} = 2 \end{aligned}$$

(2)

$$\begin{aligned} &\lim_{x \rightarrow 0} \frac{\sin x - x - x^2}{\cos x - 1} \\ &= \lim_{x \rightarrow 0} \frac{(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots) - x - x^2}{(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots) - 1} \\ &= \lim_{x \rightarrow 0} \frac{-x^2 + \dots}{-\frac{x^2}{2!} + \dots} \\ &= \lim_{x \rightarrow 0} \frac{-1 + \dots}{-\frac{1}{2!} - \dots} = \frac{-1}{-\frac{1}{2!}} = 2 \end{aligned}$$