M242 Q10(b) (S. Zhang) (8 points). Name:______________

1. Find the absolute convergence, conditional convergence and divergence by two methods:

$$\sum_{n=1}^{\infty} 2^{n+2}$$

   (a) Root test or ratio test;

   (b) Integral test or comparison test, or divergence test.

   - **ans:** (1) Root test

$$|a_n|^{1/n} = 2 \cdot 2^{2/n} \to 2 \cdot 2^0 = 2 > 1$$

The series diverges by the root test.

If we use the ratio test: +

$$\frac{|a_{n+1}|}{|a_n|} = \frac{2^{n+3}}{2^{n+2}} = 2 > 1$$

The series diverges by the ratio test.

(2) Integral test

$$\int_1^{\infty} 2^{x+2} dx = \left(\frac{2^{x+2}}{\ln 2} \right)_1^{\infty} = \infty$$

Both the series and the integral diverge.

If we use the comparison test, we need to compare it with an r-series (itself is an r-series too)

$$2^{n+2} > 2^n$$

$\sum 2^n$ diverges ($r = 2 > 1$, r-test). By the comparison test, (the bigger one) $\sum 2^{n+2}$ diverges.

If we use the divergence test,

$$a_n = 2^{n+2} \to 2^{\infty} = \infty \neq 0$$

By the divergence test, $\sum 2^{n+2}$ diverges.