M242 Q10(a) (S. Zhang) (8 points). Name:______________

1. Find the absolute convergence, conditional convergence and divergence by two methods:

$$\sum_{n=1}^{\infty} 2ne^{-n^2}$$

   (a) Root test or ratio test;

   (b) Integral test or comparison test.

   - **ans:** (1) Root test

$$|a_n|^{1/n} = 2^{1/n} n^{1/n} e^{-n} \to 2^0 \cdot 1 \cdot 0 = 0 < 1$$

The series converges absolutely by the root test. Here we may need to use L'H and take log to find

$$\lim_{n\to\infty} n^{1/n} = e^{\lim \frac{\ln n}{n}} = e^{\lim \frac{1/n}{1}} = e^0 = 1.$$

If we use the ratio test: +

$$\frac{|a_{n+1}|}{|a_n|} = \frac{2(n+1)e^{-(n+1)^2}}{2ne^{-n^2}} = (1 + \frac{1}{n})e^{-2n-1}$$
$$\to 1 \cdot e^{-\infty} = 1 \cdot 0 = 0 < 1$$

The series converges absolutely by the ratio test.

(2) Integral test

$$\int_1^{\infty} 2xe^{-x^2} dx = (-e^{-x^2})_1^{\infty} = \frac{1}{e} < \infty$$

Both the series and the integral converge. We have a positive series. So it converges absolutely.