M242 Q9(c) (S. Zhang) (8 points). Name:______________

1. (1) Show the convergence by the integral test.
   (2) Find the sum by the telescoping series method.

$$\sum_{n=1}^{\infty} \frac{1}{n^2+3n+2}$$

- **ans:** (1)

$$x^2+3x+2 = (x+1)(x+2)$$
$$\frac{1}{x^2+3x+2} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$\begin{aligned}\int_1^{\infty} \frac{dx}{x^2+3x+2} &= \int \frac{dx}{(x+1)(x+2)} \\ &= \int (\frac{1}{x+1} - \frac{1}{x+2})dx \\ &= \ln \frac{x+1}{x+2}|_1^{\infty} = -\ln\frac{2}{3} = 0.405\end{aligned}$$

Yes, both the integral and series converge.

(2)

$$\frac{1}{n^2+3n+2} = \frac{1}{n+1} - \frac{1}{n+2}$$

$$\begin{aligned}s_n =& \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + ... + \frac{1}{n+1} - \frac{1}{n+2} \\ =& \frac{1}{2} - \frac{1}{n+2} \\ \to& \frac{1}{2} = 0.5\end{aligned}$$

Yes, as the function is decreasing, the area of rectangles is a little bigger than the area under the curve (the right point rule).