

M242 Q9(b) (S. Zhang) (8 points). Name: _____

- (1) Show the convergence by the integral test.
- (2) Find the sum by the telescoping series method.

$$\sum_{n=1}^{\infty} \frac{2}{n^2 + 4n + 3}$$

• **ans:** (1)

$$\begin{aligned}x^2 + 4x + 3 &= (x + 1)(x + 3) \\ \frac{1}{x^2 + 4x + 3} &= \frac{A}{x + 1} + \frac{B}{x + 3}\end{aligned}$$

$$\begin{aligned}\int_1^{\infty} \frac{2dx}{x^2 + 4x + 3} &= \int \frac{2dx}{(x + 1)(x + 3)} \\ &= \int \left(\frac{1}{x + 1} - \frac{1}{x + 3} \right) dx \\ &= \ln \frac{x + 1}{x + 3} \Big|_1^{\infty} = -\ln \frac{1}{2} = 0.693\end{aligned}$$

Yes, both the integral and series converge.

(2)

$$\frac{2}{n^2 + 4n + 3} = \frac{1}{n + 1} - \frac{1}{n + 3}$$

$$\begin{aligned}s_n &= \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \dots + \frac{1}{n + 1} - \frac{1}{n + 3} \\ &= \frac{1}{2} + \frac{1}{3} - \frac{1}{n + 2} - \frac{1}{n + 3} \\ &\rightarrow \frac{1}{2} + \frac{1}{3} = \frac{5}{6} = 0.8333\end{aligned}$$

Yes, as the function is decreasing, the area of rectangles is a little bigger than the area under the curve (the right point rule).