M242 Q9(a) (S. Zhang) (8 points). Name:______________

1. (1) Show the convergence by the integral test.
   (2) Find the sum by the telescoping series method.

$$\sum_{n=1}^{\infty} \frac{1}{n^2+5n+6}$$

- **ans:** (1)

$$x^2+5x+6 = (x+2)(x+3)$$
$$\frac{1}{x^2+5x+6} = \frac{A}{x+2} + \frac{B}{x+3}$$

$$\begin{aligned}\int_1^{\infty} \frac{dx}{x^2+5x+6} &= \int \frac{2dx}{(x+1)(x+3)} \\ &= \int (\frac{1}{x+2} - \frac{1}{x+3})dx \\ &= \ln \frac{x+2}{x+3}|_1^{\infty} = -\ln\frac{3}{4} = 0.287\end{aligned}$$

Yes, both the integral and series converge.

(2)

$$\frac{1}{n^2+5n+6} = \frac{1}{n+2} - \frac{1}{n+3}$$

$$\begin{aligned}s_n =& \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + ... + \frac{1}{n+2} - \frac{1}{n+3} \\ =& \frac{1}{3} - \frac{1}{n+3} \\ \to& \frac{1}{3} = 0.3333\end{aligned}$$

Yes, as the function is decreasing, the area of rectangles is a little bigger than the area under the curve (the right point rule).