M242 Q8(b) (S. Zhang) (8 points). Name:______________

1. Show the convergence for both the sequence and the series by finding the limit or the sum:

$$\frac{2}{1\cdot 3}, \frac{2}{2\cdot 4}, \frac{2}{3\cdot 5}, \ldots, \frac{2}{n(n+2)}, \ldots$$
$$\frac{2}{1\cdot 3} + \frac{2}{2\cdot 4} + \frac{2}{3\cdot 5} + \ldots + \frac{2}{n(n+2)} + \ldots$$

$$\lim_{n\to\infty} a_n, \quad \sum_{n=1}^{\infty} a_n$$

where

$$a_n = \frac{2}{n(n+2)}.$$

- **ans:** (a) The sequence converges to 0:

$$\lim_{n\to\infty} \frac{2}{n(n+2)} = \frac{2}{\infty} = 0.$$

(b) The series converges too, by the telescoping series computation.

$$\frac{2}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2}$$

$$2 = A(n+2) + Bn$$
$$n = 0,\ 2 = 2A$$
$$n = -2,\ 2 = -2B$$

$$\frac{2}{n(n+2)} = \frac{1}{n} - \frac{1}{n+2}$$
$$S_n = \frac{1}{1} - \frac{1}{3} +$$
$$\frac{1}{2} - \frac{1}{4} +$$
$$\frac{1}{3} - \frac{1}{5} +$$
$$\cdots + \frac{1}{n} - \frac{1}{n+2}$$

$$= \frac{1}{1} + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}$$
$$\sum_{n=1}^{\infty} \frac{2}{n(n+2)} = \lim_{n\to\infty} S_n = \frac{3}{2}$$