

M242 Q8(a) (S. Zhang) (8 points). Name: _____

1. Show the convergence for both the sequence and the series by finding the limit or the sum:

$$\frac{2}{2 \cdot 3}, \frac{2}{3 \cdot 4}, \frac{2}{4 \cdot 5}, \dots, \frac{2}{(n+1)(n+2)}, \dots$$
$$\frac{2}{2 \cdot 3} + \frac{2}{3 \cdot 4} + \frac{2}{4 \cdot 5} + \dots + \frac{2}{(n+1)(n+2)} + \dots$$

$$\lim_{n \rightarrow \infty} a_n, \quad \sum_{n=1}^{\infty} a_n$$

where

$$a_n = \frac{2}{(n+1)(n+2)}.$$

• **ans:** (a) The sequence converges to 0:

$$\lim_{n \rightarrow \infty} \frac{2}{(n+1)(n+2)} = \frac{2}{\infty} = 0.$$

(b) The series converges too, by the telescoping series computation.

$$\frac{2}{(n+1)(n+2)} = \frac{A}{n+1} + \frac{B}{n+2}$$

$$2 = A(n+2) + B(n+1)$$

$$n = -1, 2 = A$$

$$n = -2, 2 = -B$$

$$\frac{2}{(n+1)(n+2)} = \frac{2}{n+1} - \frac{2}{n+2}$$

$$S_n = \frac{2}{2} - \frac{2}{3} +$$

$$\frac{2}{3} - \frac{2}{4} +$$

$$\frac{2}{4} - \frac{2}{5} +$$

$$\dots + \frac{2}{n+1} - \frac{2}{n+2}$$

$$= \frac{2}{2} - \frac{2}{n+2}$$

$$\sum_{n=1}^{\infty} \frac{2}{(n+1)(n+2)} = \lim_{n \rightarrow \infty} S_n = 1$$