

M242 Q7(b) (S. Zhang) (8 points). Name: _____

1. We can show (not required) that the sequence

$$a_1 = 1$$
$$a_{n+1} = \sqrt{8 + \frac{a_n}{3}}, \quad n = 1, 2, \dots$$

is increasing and bounded above. Therefore it is convergent. Find the limit.

• **ans:** Take limit on the equation

$$L^2 = 8 + \frac{L}{3}$$
$$3L^2 - L - 24 = 0,$$
$$(3L + 8)(L - 3) = 0, \quad L = -\frac{8}{3}, 3$$

Since a_n starts with a positive number 1 and increases, the limit would be positive and bigger than 1. The correct answer for the limit is 3. We can check first few terms:

$$1, 2.8868, 2.9937, 2.9997, \dots$$