

M242 Q6(a) (S. Zhang) (8 points). Name: _____

1. Find

$$\int_0^2 \frac{x}{\sqrt{|x-1|}} dx$$

(hint: $\sqrt{|x-1|} = \sqrt{1-x}$ when $x \leq 1$.)

• **ans:**

$$\int_0^2 \frac{x}{\sqrt{|x-1|}} dx = \int_0^1 \frac{x}{\sqrt{1-x}} dx + \int_1^2 \frac{x}{\sqrt{x-1}} dx$$

Let

$$\begin{array}{ll} u = \sqrt{1-x} & w = \sqrt{x-1} \\ u^2 = 1-x & w^2 = x-1 \\ 2udu = -dx & 2wdw = dx \end{array}$$

$$\begin{aligned} \int_0^2 \frac{x}{\sqrt{|x-1|}} dx &= \int \frac{(1-u^2)(-2udu)}{u} + \int \frac{(w^2+1)2wdw}{w} \\ &= (-2u + \frac{2}{3}u^3) + (\frac{2}{3}w^3 + 2w) \\ &= (-2\sqrt{1-x} + \frac{2}{3}(1-x)^{3/2})_0^1 + (\frac{2}{3}(x-1)^3 + 2\sqrt{x-1})_1^2 \\ &= (0 - (-2 + \frac{2}{3})) + ((\frac{2}{3} + 2) - 0) \\ &= 4 \end{aligned}$$