

M242 Q2(c) (S. Zhang) . Name: _____

1. Find the limit

$$(1) \lim_{x \rightarrow \infty} \left(1 - \frac{3}{x}\right)^{2x}$$

$$(2) \lim_{x \rightarrow 1} \left(1 - \frac{3}{x}\right)^{2x}$$

• **ans:**

limit((1-3/x)^(2*x), x =infinity)

exp(-6)

(1) It is of type 1^∞ . Before doing it, we know the answer is between 0 and 1 since the base is less than 1.

$$F = \left(1 - \frac{3}{x}\right)^{2x}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \ln F &= \lim_{x \rightarrow \infty} \frac{2 \ln(1 - 3/x)}{1/x} \\ &= \lim_{x \rightarrow \infty} \frac{2(1 - 3/x)^{-1}(-3x^{-2})}{-x^{-2}} \\ &= \lim_{x \rightarrow \infty} \frac{2(-3)}{1 - 3/x} \\ &= -6 \end{aligned}$$

$$\lim_{x \rightarrow \infty} \left(1 - \frac{3}{x}\right)^{2x} = e^{\lim_{x \rightarrow \infty} \ln F} = e^{-6}$$

(2) It is not an indeterminate form:

$$\lim_{x \rightarrow 1} \left(1 - \frac{3}{x}\right)^{2x} = (-2)^2 = 4$$