

M242 Q2(b) (S. Zhang) . Name: _____

1. Find the limit

$$(1) \lim_{x \rightarrow 0^+} (1 + \sin x)^{2/x}$$

$$(2) \lim_{x \rightarrow \pi} (1 + \sin x)^{2/x}$$

• **ans:**

limit((1+sin(x))^(2/x), x =0)
exp(2)

(1) It is of type 1^∞ . Before doing it, we know the answer is between 1 and ∞ since the base is bigger than 1.

$$F = (1 + \sin x)^{2/x}$$

$$\begin{aligned} \lim_{x \rightarrow 0^+} \ln F &= \lim_{x \rightarrow 0^+} \frac{2 \ln(1 + \sin x)}{x} \\ &= \lim_{x \rightarrow 0^+} \frac{2 \left(\frac{\cos x}{1 + \sin x} \right)}{1} \\ &= 2 \end{aligned}$$

$$\lim_{x \rightarrow 0^+} (1 + \sin x)^{2/x} = e^{\lim_{x \rightarrow 0^+} \ln F} = e^2$$

(2) It is not an indeterminate form:

$$\lim_{x \rightarrow \pi} (1 + \sin x)^{2/x} = 1^{2/\pi} = 1$$