

M242 Q2(a) (S. Zhang) . Name: _____

1. Find the limit

$$(1) \lim_{x \rightarrow \infty} (\ln x)^{3/x}$$

$$(2) \lim_{x \rightarrow 1} (\ln x)^{3/x}$$

• ans:

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limit((ln(x))^(3/x), x=infinity)
1
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(1) It is of type ∞^0 . Before doing it, we know the answer is between 1 and ∞ since the base is bigger than 1.

$$F = (\ln x)^{3/x}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \ln F &= \lim_{x \rightarrow \infty} \frac{3 \ln(\ln x)}{x} \\ &= \lim_{x \rightarrow \infty} \frac{3 \left(\frac{1}{x \ln x} \right)}{1} \\ &= \lim_{x \rightarrow \infty} \frac{3 \left(\frac{1}{x} \right)}{\ln x} \\ &= \lim_{x \rightarrow \infty} \frac{3 \left(-\frac{1}{x^2} \right)}{1/x} \\ &= \lim_{x \rightarrow \infty} \frac{-3}{x} \\ &= 0 \end{aligned}$$

$$\lim_{x \rightarrow \infty} (\ln x)^{3/x} = e^{\lim_{x \rightarrow \infty} \ln F} = e^0 = 1$$

(2) It is not an indeterminate form:

$$\lim_{x \rightarrow 1} (\ln x)^{3/x} = 0^3 = 0$$