

**M242 Hw12** (S. Zhang) 10.2: 5-7, 12-13, 16-18, 29-32, 41-42, 51-52, 59-60  
 10.3: 4-5, 7-9, 15-18, 24-26, 32-34, 61-64. .

1. (10.2:7) Find the tangent line by (1) directly; (2) by 52.41 eliminating  $t$ .

$$\begin{aligned}x &= 1 + \ln t \\ y &= t^2 + 2, \quad (1, 3)\end{aligned}$$

- **ans:** (1) using the first equation and point (1, 3)

$$1 = 1 + \ln t, \quad t = 1$$

$$\frac{dy}{dx} = \frac{y'(t)dt}{x'(t)dt} = \frac{y'}{x'} = \frac{2t}{1/t}$$

At  $t = 1$ ,

$$\frac{dy}{dx} = 2$$

The tangent line at point (1, 3):

$$\frac{y - 3}{x - 1} = 2$$

(2) By equation 1,

$$t = e^{x-1}$$

By equation 2,

$$y = t^2 + 2 = e^{2x-2} + 2$$

$$\frac{dy}{dx} = 2e^{2x-2}$$

$$x = 1, \Rightarrow \frac{dy}{dx} = 2e^0 = 2$$

The tangent line at point (1, 3):

$$\frac{y - 3}{x - 1} = 2$$

2. (10.2:11) Find  $dy/dx$ ,  $d^2y/dx^2$ , and  $t$  range that the 52.43 curve is concave up.

$$\begin{aligned}x &= t^3 - 12t \\ y &= t^2 - 1\end{aligned}$$

- **ans:**

$$\frac{dy}{dx} = \frac{y'(t)dt}{x'(t)dt} = \frac{y'}{x'} = \frac{2t}{3t^2 - 12}$$

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{d(dy/dx)}{dx} = \frac{(dy/dx)'}{x'} \\ &= \frac{(3t^2-12)2-2t(6t)}{(3t^2-12)^2} \\ &= \frac{3t^2 - 12}{3t^2 - 12} \\ &= \frac{-2(t^2 + 4)}{9(3t^2 - 12)^3}\end{aligned}$$

Concave up when  $\frac{d^2y}{dx^2} > 0$ . So

$$\begin{aligned}(3t^2 - 12)^3 &< 0 \\ 3t^2 - 12 &< 0 \\ t^2 &< 4 \\ t &< 2 \text{ and } t > -2 \\ -2 &< t < 2\end{aligned}$$

3. (10.2:41) Find the arc length 52.45

$$\begin{aligned}x &= 1 + 3t^2 \\ y &= 4 + 2t^3, \quad 0 \leq t \leq 1\end{aligned}$$

- **ans:**

arc length:

$$\begin{aligned}L &= \int \sqrt{dx^2 + dy^2} = \int \sqrt{(x')^2 + (y')^2} dt \\ &= \int_0^1 \sqrt{(6t)^2 + (6t^2)^2} dt \\ &= 6 \int_0^1 t \sqrt{1 + t^2} dt = 6 \int \sqrt{1 + t^2} \frac{1}{2} d(1 + t^2) \\ &= 3 \left. \frac{(1 + t^2)^{3/2}}{3/2} \right|_0^1 = 2(2\sqrt{2} - 1)\end{aligned}$$

1. (10.3:5(a)) Find the polar coordinates of (2, -2) 53.41  
 (1)  $r > 0, 0 \leq \theta < 2\pi$ ;  
 (2)  $r < 0, 0 \leq \theta < 2\pi$ ;

- **ans:**

$$\begin{aligned}r^2 &= x^2 + y^2 = 8 \\ \tan \theta &= \frac{y}{x} = -1\end{aligned}$$

The point (2, -2) is in the 4-th quadrant.

(1)

$$r = 2\sqrt{2}, \quad \theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

(2) We need to point the direction into the 2-nd quadrant:

$$r = -2\sqrt{2}, \quad \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

2. (10.3:18) Identify the curve by finding a Cartesian equation for the curve

$$r = 2 \sin \theta + 2 \cos \theta$$

• **ans:**

$$r = 2 \sin \theta + 2 \cos \theta = \frac{2y}{r} + \frac{2x}{r}$$

$$r^2 = 2y + 2x$$

$$x^2 + y^2 = 2y + 2x$$

$$(x^2 - 2x + 1) + (y^2 - 2y + 1) = 1 + 1$$

$$(x - 1)^2 + (y - 1)^2 = (\sqrt{2})^2$$

A circle of radius  $\sqrt{2}$  centered at  $(1, 1)$ .