M242 Hw11 (S. Zhang) 11.11: 5-6, 15(ab)-18(ab) 10.1: 1, 6-8, 11-13, 19 .

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1. (11.11:15) Find the Taylor polynomial $T_n(x)$ for the
41.41 function f at the number a. And bound the error for $T_n(x)$ by R_n on the given interval .

$$f(x) = x^{2/3},\ a = 1,\ n = 3,\ x \in [0.8, 1.2]$$

- **ans:**

$$\begin{aligned} f(x) &= x^{2/3}, & f(1) &= 1 \\ f'(x) &= \frac{2}{3}x^{-1/3}, & f'(a) &= \frac{2}{3} \\ f''(x) &= -\frac{2}{9}x^{-4/3}, & f''(a) &= -\frac{2}{9} \\ f'''(x) &= \frac{8}{27}x^{-7/3}, & f''(a) &= \frac{8}{27} \end{aligned}$$

$$f^{(4)}(z) = -\frac{56}{81}z^{-10/3}$$

$$\begin{aligned} T_3 &= f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 \\ &+ \frac{f'''(a)}{3!}(x-a)^3 \\ &= 1 + \frac{2}{3}(x-1) - \frac{1}{9}(x-1)^2 + \frac{4}{81}(x-1)^3 \\ R_3(x) &= \frac{f^{(4)}(z)}{4!}(x-a)^4 = -\frac{56/81}{4!}z^{-10/3}(x-1)^4 \end{aligned}$$

$$\begin{aligned} \max_{x\in[0.8,1.2]} |R_3(x)| \leq & \frac{56/81}{24}\frac{1}{0.8^{10/3}}|1.2-1|^4 \\ =& 0.000096 \end{aligned}$$

Rough checking:

$$\begin{aligned} f(0.8) - T_3(0.8) &= -0.0000532 \\ f(1.2) - T_3(1.2) &= -0.0000407 \end{aligned}$$

2. (11.11:18) Find the Taylor polynomial $T_n(x)$ for the
41.42 function f at the number a. And bound the error for $T_n(x)$ by R_n on the given interval .

$$f(x) = \ln(1+2x),\ a = 1,\ n = 3,\ x \in [0.5, 1.5]$$

- **ans:**

$$\begin{aligned} f(x) &= \ln(1+2x), & f(1) &= \ln 3 \\ f'(x) &= 2(1+2x)^{-1}, & f'(a) &= \frac{2}{3} \\ f''(x) &= -4(1+2x)^{-2}, & f''(a) &= -\frac{4}{9} \\ f'''(x) &= 16(1+2x)^{-3}, & f''(a) &= \frac{16}{27} \end{aligned}$$

$$\frac{f^{(4)}(z)}{4!} = -4(1+2z)^{-4}$$

$$\begin{aligned} T_3 &= f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 \\ &+ \frac{f'''(a)}{3!}(x-a)^3 \\ &= \ln 3 + \frac{2}{3}(x-1) - \frac{2}{9}(x-1)^2 + \frac{8}{81}(x-1)^3 \\ R_3(x) &= \frac{f^{(4)}(z)}{4!}(x-a)^4 = -4(1+2z)^{-4}(x-1)^4 \end{aligned}$$

$$\begin{aligned} \max_{x\in[0.5,1.5]} |R_3(x)| \leq & 4(1+2\cdot 0.5)^{-4}|1.5-1|^3 \\ =& 0.015625 \end{aligned}$$

Rough checking:

$$\begin{aligned} f(0.5) - T_3(0.5) &= -0.00423 \\ f(1.5) - T_3(1.5) &= -0.00244 \end{aligned}$$

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1. (10.1:8) Sketch curve, and indicate the direction when
51.41 t increases. And find the Cartesian equation.

$$\begin{aligned} x =& 1 + 3t \\ y =& 2 - t^2 \end{aligned}$$

- **ans:**

t	-3	-2	-1	0	1	2	3
x	-8	-5	-2	1	4	7	10
y	-7	-2	1	2	1	-2	-7

(A graph to be inserted.)

By the frist equation ,

$$t = \frac{x-1}{3}$$

By the second equation,

$$y = 2 - \frac{1}{9}(x-1)^2$$

2. (10.1:19) Describe the motion.
51.43

$$\begin{aligned} x =& 3 + 2\cos t \\ y =& 1 + 2\sin t,\ \frac{\pi}{2} \leq t \leq \frac{3\pi}{2} \end{aligned}$$

- **ans:** To eliminate t

$$\begin{aligned} \frac{x-3}{2} &= \cos t \\ \frac{y-1}{2} &= \sin t \end{aligned}$$

Square two equations and add them up

$$\left(\frac{x-3}{2}\right)^2 + \left(\frac{y-1}{2}\right)^2 = 1$$

$$(x-3)^2 + (y-1)^2 = 2^2$$

Part (left half) of the circle centered at $(3, 1)$ with radius $r = 2$.

(A graph to be inserted.)