M242 Hw5 (S. Zhang)
7.7: a1-a4.
7.8: 6-8,13-15,20-23,28-33,49-51,78-79.

1. Compute by the trapezoidal rule with $h = 2$ and $h = 1$ (2 methods) and find the error bound each time

$$\int_1^4 8x^2\,dx$$

- **ans:** Exact solution 168.

$$h = 3, \quad x_0 = 1, x_1 = 4, \ n = 1$$

$$T_{h=3} = h(\frac{1}{2}f(x_0) + \frac{1}{2}f(x_1))$$
$$= 204$$

$$K_2 = \max_{a\le x\le b} |f''| = 16(1) = 16$$

$$|E_T| \le K_2\frac{(b-a)^3}{12n^2}$$
$$= 16(3)^3/(12\cdot 1^2) = 36$$

Actural error is $168 - 204 = -36$.

$$h = 1.5, \quad x_0 = 1, x_1 = 2.5, x_2 = 4, \ n = 2$$

$$T_{h=1.5} = \frac{3}{2}(\frac{1}{2}f(x_0) + f(x_1) + \frac{1}{2}f(x_2))$$
$$= 177$$

$$|E_T| \le K_2\frac{(b-a)^3}{12n^2}$$
$$= 16(3)^3/(12\cdot 2^2) = 9$$

Actual error is $168 - 177 = -9$.

Method 2: (Save more function evalutaions than the mid-point rule)

$$T_{h=1.5} = \frac{1}{2}T_{h=3} + h(f(2.5))$$
$$= 204/2 + (3/2)(2(5)^2) = 354/2 = 177.$$

2. Given

$$\int_{\sin 8}^{2+\sin 8} f(x)\,dx, \ R_{h=2} = 0, \ L_{h=2} = 512; \ M_{h=2} = 32,$$

Find

$$T_{h=2}, T_{h=1}, S_{h=1}.$$

- **ans:**

$$T_h = \frac{1}{2}R_h + \frac{1}{2}L_h$$

$$T_h = \frac{1}{2}T_{2h} + \frac{1}{2}M_{2h}$$

$$S_h = \frac{1}{3}T_{2h} + \frac{2}{3}M_{2h}$$

$$T_{h=2} = \frac{1}{2}R_{h=2} + \frac{1}{2}L_{h=2} = 256$$

$$T_{h=1} = \frac{1}{2}T_{h=2} + \frac{1}{2}M_{h=2} = 144$$

$$S_{h=1} = \frac{1}{3}T_{h=2} + \frac{2}{3}M_{h=2} = \frac{320}{3}$$

---

1. (7.8:8) Find

$$\int_0^\infty \frac{xdx}{(x^2+2)^2}$$

- **ans:**

$$u = x^2 + 2, \ du = 2xdx$$

$$\int_0^\infty \frac{xdx}{(x^2+2)^2}$$
$$= \int \frac{du/2}{u^2}$$
$$= -\frac{1}{2}u^{-1} = \left(-\frac{1}{2}(x^2+2)^{-1}\right)_0^\infty$$
$$= 0 - (-\frac{1}{2}(2^{-1})) = \frac{1}{4}$$

2. (7.8:31) Find

$$\int_{-2}^3 \frac{dx}{x^4}$$

- **ans:**

$$\int_{-2}^3 \frac{dx}{x^4}$$
$$= \int_{-2}^0 + \int_0^3$$
$$= \left(-\frac{1}{3}x^{-3}\right)_{-2}^0 + \left(-\frac{1}{3}x^{-3}\right)_0^3$$
$$= (\infty - \frac{1}{24}) + (\infty - \frac{1}{81} + \infty)$$
$$= \infty$$

3. (7.8:50) Find out the convergence:

$$\int_1^{\infty} \frac{2+e^{-x}}{x} dx$$

- **ans:**

$$f = \frac{2+e^{-x}}{x}$$
$$g = \frac{1}{x}$$
$$f(x) \geq g(x), \ 1 \leq x \leq \infty$$

$$\int_1^{\infty} g = (\ln x)_1^{\infty} = \infty$$

The (smaller) integral diverges ⇒ the (bigger) integral diverges.

$$\int_1^{\infty} \frac{2+e^{-x}}{x} dx = \infty$$