M242 Hw3 (S. Zhang)
7.1: 2-4, 7-10, 15, 17-18, 22-23, 27-29, 33-34, 47-48, 51-52.
7.2: 3-5, 8-11, 13-14, 21-24, 29-30, 33-34, 43-46, 57, 62 .

1. (7.1:10) Find

11.31

$$\int \sin^{-1} x\, dx$$

• **ans:** Since there is no other way, and there is no choice either, we give a try, letting

$$u = \sin^{-1} x, \qquad dv = dx$$
$$du = \frac{1}{\sqrt{1-x^2}}dx, \qquad v = x$$

$$\int udv = uv - \int vdu$$

$$\int \sin^{-1} x\, dx = x \sin^{-1} x - \int \frac{xdx}{\sqrt{1-x^2}}$$

For the second term, it is easy to see a variable substitution

$$w = \sqrt{1-x^2},\ dw = \frac{xdx}{\sqrt{1-x^2}}$$
$$\int \frac{xdx}{\sqrt{1-x^2}} = \int dw = \sqrt{1-x^2} + C$$

Together,

$$\int \sin^{-1} x\, dx$$
$$= x \sin^{-1} x - \sqrt{1-x^2} + C$$

2. (7.1:22) Find

11.33

$$\int_4^9 \frac{\ln y}{\sqrt{y}} dy$$

• **ans:** The idea is to make u' "simpler than" u while v' and v are about the "same".

Here we have to let $u = \ln y$ because if this function is part of dv, it is hard to recover v. So we are forced to do so.

$$u = \ln y, \qquad dv = \frac{1}{\sqrt{y}}\, dy$$
$$du = \frac{1}{y}dy, \qquad v = 2\sqrt{y}$$

$$\int udv = uv - \int vdu$$

$$\int \frac{\ln y}{\sqrt{y}} dx = 2\sqrt{y}\ln y - \int 2\sqrt{y}\frac{1}{y}dy$$
$$= 2\sqrt{y}\ln y - \int 2\frac{1}{\sqrt{y}}dy$$
$$= 2\sqrt{y}\ln y - 2\sqrt{y} + C$$
$$\int_4^9 \frac{\ln y}{\sqrt{y}} dx = (2\sqrt{y}\ln y - 2\sqrt{y})_4^9$$
$$= 12\ln 3 - 8\ln 2 - 4$$

3. (7.1:22) Find (changing variable plus integration by
11.35 parts)

$$\int t^3 e^{-t^2} dt$$

• **ans:** The idea is to make u' "simpler than" u while v' and v are about the "same". So we would let dv be the part involving e^{-t^2}. But to find the anti-derivative to recover v, we need an extra term t as $(e^{-t^2})' = -2te^{-t^2}$, We can do it by changing variable first.

$$x = t^2,\ dx = 2tdt$$

$$\int t^3 e^{-t^2} dt = \frac{1}{2}\int xe^{-x}dx$$

So it is easy to see:

$$u = x, \qquad dv = e^{-x}dx$$
$$du = dx, \qquad v = -e^{-x}$$

$$\int udv = uv - \int vdu$$

$$\int xe^{-x}dx = -xe^{-x} - \int -e^{-x}dx$$
$$= -xe^{-x} - e^{-x} + C$$

$$\int t^3 e^{-t^2} dt = -\frac{1}{2}(x+1)e^{-x} + C$$
$$= -\frac{1}{2}(t^2+1)e^{-t^2} + C$$

Well, we do not need to do this variable substitution if we can see the function clearly. We can let directly

$$u = t^2, \qquad dv = te^{-t^2}dt$$
$$du = 2tdx, \qquad v = -\frac{1}{2}e^{-t^2}$$

$$\int udv = uv - \int vdu$$

$$\begin{aligned}\int t^3 e^{-t^2} dt &= -\frac{1}{2}t^2 e^{-t^2} - (-\int 2t\frac{1}{2}e^{-t^2}) \\ &= -\frac{1}{2}t^2 e^{-t^2} + \int te^{-t^2} dt \\ &= -\frac{1}{2}t^2 e^{-t^2} - \frac{1}{2}e^{-t^2} + C\end{aligned}$$

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1. (7.2:8) Find

13.27

$$\int_0^{\pi/2} \sin^2(2\theta)d\theta.$$

- **ans:** Double angle formula:

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\begin{aligned}\int_0^{\pi/2} \sin^2(2\theta)d\theta &= \int \frac{1-\cos 4\theta}{2} d\theta \\ &= \left(\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta\right)_0^{\pi/2} \\ &= \left(\frac{1}{2}\frac{\pi}{2} - \frac{1}{8}\sin 2\pi\right) - (0-0) = \frac{\pi}{4}.\end{aligned}$$

2. Find the area of the region bounded by the given curves:

13.28

$$y = \sin^2 x,\ y = \cos^2 x,\ -\frac{\pi}{4} \le x \le \frac{\pi}{4}$$

- **ans:**

$$A = \int_{-\pi/4}^{\pi/4} (\cos^2 x - \sin^2 x)dx$$

using double angle formulas,

$$\begin{aligned}A &= \int_{-\pi/4}^{\pi/4} \cos 2x dx \\ &= \left(\frac{1}{2}\sin 2x\right)_{-\pi/4}^{\pi/4} \\ &= \frac{1}{2} + \frac{1}{2} = 1\end{aligned}$$