

M242 Hw1 (S. Zhang)

4.8: 5-7, 11-12, 35-36

4.4: 5-7, 10-11, 24-26, 40-42, 49-55

6.1: 5-10, 31-32, 50-51.

1. (4.8:5)

Use Newton's method to find x_3 :

$$x^3 + 2x - 4 = 0, \quad x_1 = 1$$

• **ans:**

$$f(x) = x^3 + 2x - 4$$

$$f'(x) = 3x^2 + 2$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 1,$$

$$f(x_1) = -1$$

$$f'(x_1) = 5$$

$$x_2 = x_1 - f(x_1)/f'(x_1) = 1.2,$$

$$f(x_2) = 0.128$$

$$f'(x_2) = 6.32$$

$$x_3 = 1.1797$$

x_i	$f(x_i)$	$f'(x_i)$
1	-1	5
1.2	0.128	6.32
1.1797		

The root is 1.179509025 by Maple fsolve.

$$f:=x \rightarrow x^3+2*x-4: \quad g:=x \rightarrow 3*x^2+2:$$

$$t:=1: \quad x1:=t:$$

$$\text{evalf}([t,f(t),g(t)]); \quad t:=t-f(t)/g(t):$$

$$x2:=t:$$

$$\text{evalf}([t,f(t),g(t)]); \quad t:=t-f(t)/g(t):$$

$$x3:=t:$$

$$\text{evalf}([t,f(t),g(t)]);$$

$$\text{fsolve}(f(x) = 0, x)$$

2. (4.8:12)

Use Newton's method to approximate the given number to 6 decimal places:

$$\sqrt[100]{100}$$

• **ans:** Let $x = \sqrt[100]{100}$ Then

$$x^{100} = 100$$

$$f(x) = x^{100} - 100$$

$$f'(x) = 100x^{99}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

If we choose $x_1 = 1$ or some other numbers, the iteration would diverge. We need to choose the initial x_1 close enough to the root. We (using calculator to get 1.047128548, "cheating") choose $x_1 = 1.05$:

$$x_1 = 1.05,$$

$$f(x_1) = 31.5012578$$

$$f'(x_1) = 12523.9293$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.047484715, \quad f(x_2) = 3.4592776$$

$$f'(x_2) = 9876.92$$

$$x_3 = 1.047134477$$

x_i	$f(x_i)$	$f'(x_i)$
1.05	31.5012578	12523.92
1.047484715	3.4592776	9876.92
1.047134477	0.0566369	9555.28
1.047128550	0.0000186	

$$f:=x \rightarrow x^{100}-100: \quad g:=x \rightarrow 100*x^{99}:$$

$$t:=1.05: \quad x1:=t:$$

$$\text{evalf}([t,f(t),g(t)]); \quad t:=t-f(t)/g(t):$$

$$x2:=t:$$

$$\text{evalf}([t,f(t),g(t)]); \quad t:=t-f(t)/g(t):$$

$$x3:=t:$$

$$\text{evalf}([t,f(t),g(t)]); \quad t:=t-f(t)/g(t):$$

1. (4.4:40)

Find the limit

$$\lim_{x \rightarrow -\infty} x^2 e^x$$

• **ans:** It is of type $\infty \cdot 0$. We need to flip one of the function downstairs. The idea is to have f' and g' simpler than the functions themselves. So we must flip e^x down.

$$\begin{aligned} \lim_{x \rightarrow -\infty} x^2 e^x &= \lim_{x \rightarrow -\infty} \frac{x^2}{e^{-x}} \\ &\stackrel{\infty/\infty}{=} \lim_{x \rightarrow -\infty} \frac{2x}{-e^{-x}} \\ &\stackrel{\infty/\infty}{=} \lim_{x \rightarrow -\infty} \frac{2}{e^{-x}} = \frac{2}{\infty} = 0 \end{aligned}$$

2. (4.4:53)

Find the limit

$$\lim_{x \rightarrow 0^+} x^{x^2}$$

- **ans:** It is of type 0^0 .

Because the base is small, less than 1, and the power is positive, the limit must be between 0 and 1. Note that $0^a = 0$ and $a^0 = 1$, for $a > 0$. Having this in mind before doing the problem.

We compute the limit of \ln first:

$$\begin{aligned} \lim_{x \rightarrow 0^+} \ln x^{x^2} &= \lim_{x \rightarrow 0^+} \frac{\ln x}{x^{-2}} \\ &\stackrel{\infty/\infty}{=} \lim_{x \rightarrow 0^+} \frac{x^{-1}}{-2x^{-3}} \\ \lim_{x \rightarrow 0^+} \frac{x^2}{-2} &= \frac{0}{-2} = 0 \end{aligned}$$

So

$$\lim_{x \rightarrow 0^+} x^{x^2} = e^0 = 1$$

3. (6.1:10)

Sketch the region. Draw a typical approximating rectangle and label its height and width. Find the area.

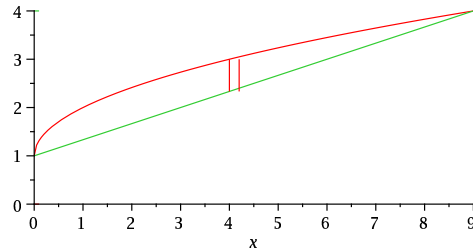
$$y = 1 + \sqrt{x}, \quad y = 1 + \frac{x}{3}$$

- **ans:** Find the intersections of the two curves:

$$1 + \sqrt{x} = 1 + \frac{x}{3}$$

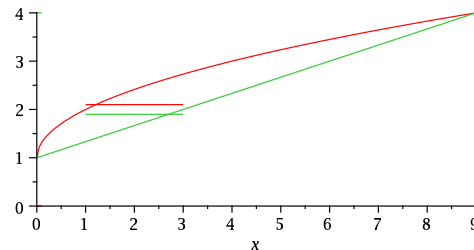
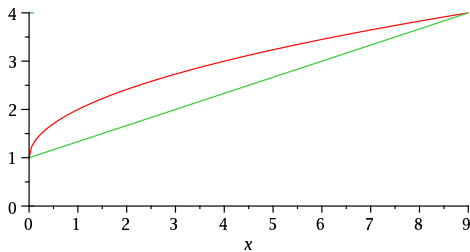
$$x = \frac{x^2}{9}, \quad x^2 - 9x = 0$$

$$x = 0, 9, \Rightarrow (0, 1), (9, 4)$$



Or we can cut the region horizontally, – **Method 2**

$$\begin{aligned} A &= \int_1^4 (x_{right} - x_{left}) dy \\ &= \int_1^4 ((3y - 3) - (y - 1)^2) dx \\ &= \left(\frac{3}{2}y^2 - 3y - \frac{1}{3}(y - 1)^3 \right)_1^4 = \frac{9}{2} \end{aligned}$$



We can find the area by cutting it vertically:

$$\begin{aligned} A &= \int_0^9 (y_{top} - y_{bottom}) dx \\ &= \int_0^9 (1 + \sqrt{x} - (1 + \frac{x}{3})) dx \\ &= \left(\frac{2}{3}x^{3/2} - \frac{x^2}{6} \right)_0^9 = \frac{9}{2} \end{aligned}$$

```
p1:=plot({0,4},x=0..0.1,
scaling=constrained);
p2:=plot({1+sqrt(x),1+x/3},x=0..9);
display({p1,p2});
p3:=implicitplot({x=4,x=4.2},
x=4..5,y=7/3..3);
display({p1,p2,p3});
p4:=plot({1.9,2.1},x=1..3);
display({p1,p2,p4});
int((3y-3)-(y-1)^2,y=1..4)
```

4. (6.1:32)

Sketch the region. Find the area.

$$\int_0^4 |\sqrt{x+2} - x| dx$$

- **ans:** Find the intersection(s) of the two curves:

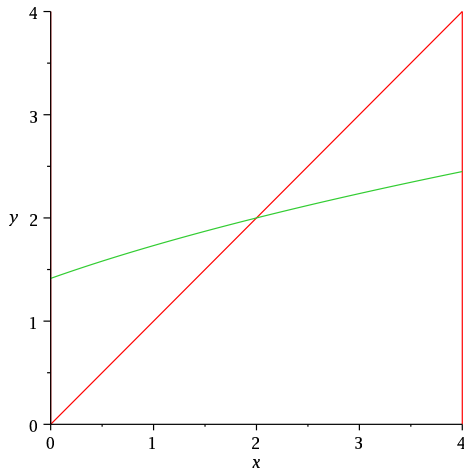
$$y = \sqrt{x+2} = x$$

$$x^2 - x - 2 = 0$$

$$x = -1, 2, \Rightarrow (2, 2)$$

The other x is outside the integral.

From the graph, we can determine how to choose a sign for the absolute function. But we can “cheat” by computing both integrals and use positive areas for them.



$$\begin{aligned} A_1 &= \int_0^2 (\sqrt{x+2} - x) dx \\ &= \left(\frac{2}{3}(x+2)^{3/2} - \frac{1}{2}x^2 \right)_0^2 = \frac{10}{3} - \frac{4\sqrt{2}}{3} \end{aligned}$$

$$\begin{aligned} A_2 &= \int_2^4 (\sqrt{x+2} - x) dx \\ &= \left(\frac{2}{3}(x+2)^{3/2} - \frac{1}{2}x^2 \right)_2^4 = -\frac{34}{3} + 4\sqrt{6} \end{aligned}$$

So

$$A = |A_1| + |A_2| = A_1 - A_2 = \frac{44}{3} - \frac{4\sqrt{2}}{3} - 4\sqrt{6}$$

```
p1:=plot({x,sqrt(x+2)},x=0..4,
scaling=constrained);
p3:=implicitplot({x=0,x=4},
x=0..4,y=0..4);
display({p1,p3});
int(sqrt(x+2)-x,x=2..4)
```