ERRATUM TO “A C1-P2 FINITE ELEMENT WITHOUT NODAL BASIS”

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Abstract. A new interpolation operator is defined, which preserves only P2 polynomials locally.

Mathematics Subject Classification. 65N30, 73C35.

Received November 24, 2019. Accepted February 17, 2020.

In [1], a finite element interpolation operator is defined and it is claimed (incorrectly) that the operator preserves the finite element functions (piecewise $P_2$ polynomials) and that consequently it preserves $P_2$ polynomials. In this note, we would change the definition of this interpolation operator, from (1) to (5) below, so that it preserves only $P_2$ polynomials.

In [1], a dual basis $\psi_i$, corresponding to a finite element nodal basis $\phi_i$ of $V_{h,0}$, is defined on a 9-square patch $M_i$ as

$$\psi_i = \frac{583704}{553687} (\phi_{S_1} + \phi_{S_3} + \phi_{S_5} + \phi_{S_9}) - \frac{970452}{553687} (\phi_{S_2} + \phi_{S_4} + \phi_{S_6} + \phi_{S_8}) + \frac{1743594}{553687} \phi_{S_5},$$  

(1)

where $S_l$, cf. Figure 1, is the local indexing of squares around square $Q_i$ ($i = S_5$). Then the local interpolation operator is defined by

$$I_h u = \sum_{i=1}^{n^2} u_i \phi_i, \quad \text{where} \quad u_i = \int_{M_i} \psi_i(x) u(x) \, dx.$$  

(2)

Lemma 3.2 of [1] mistakenly claims that

$$I_h u = u \quad \text{on} \quad Q_i \quad \text{if} \quad u = \sum_{i=1}^{n^2} u_i \phi_i \quad \text{on} \quad M_i,$$

(3)

and that consequently the interpolation preserves $P_2$ polynomials locally. Because of overlapping, one cannot have (3) but only have

$$I_h u = u \quad \text{on} \quad Q_i \quad \text{if} \quad u = \sum_{i=1}^{9} u_{S_i} \phi_{S_i} \quad \text{on} \quad N_i,$$

(4)

where $N_i$ is a patch of 25 squares around $Q_i$, cf. Figure 1. Therefore, $I_h$ does not preserve $P_2$ polynomials either.

Keywords and phrases. Differentiable finite element, quadratic element, biharmonic equation.

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We give up the preservation of all finite element $V_{h,0}$ functions. For preserving only $P_2$ polynomials locally, the dual basis function $\psi_i$ is simply defined by, replacing (1),

$$\psi_i = 0(\phi_{S_1} + \phi_{S_3} + \phi_{S_7} + \phi_{S_9}) - \frac{45}{386}(\phi_{S_2} + \phi_{S_4} + \phi_{S_6} + \phi_{S_8}) + \frac{493}{772}\phi_{S_5}. \quad (5)$$

Lemma 3.2 of [1] is replaced by the next lemma.

**Lemma 1.** If $u \in P_2(N_i)$, cf. Figure 1, then

$$I_h u = u \quad \text{on} \quad Q_i,$$

where $I_h$ is defined in (2) with $\psi_i$ defined in (5).

**Proof.** Let the lower-left corner of square $Q_{S_i}$ be $(0, 0)$, cf. Figure 1, and the grid size $h = 1$.

$$\int_{M_i} \psi_i(x, y)u(x, y) \, dx \, dy = \begin{cases} 
1/2 & \text{if } u = 1, \\
3/4 & \text{if } u = x, \quad \text{or } y, \\
1 & \text{if } u = x^2, \quad \text{or } y^2, \\
9/8 & \text{if } u = xy. 
\end{cases}$$

Combining 9 terms, with the definition of $\phi_{S_i}$, we get $I_h x^k y^l = x^k y^l$ on $Q_i$ for $k + l \leq 2$.

**Remark 1.** The rest analysis and results remain same in [1], after the above correction.

**REFERENCES**