Solutions to Homework MATH 350

Chapter 2:

2.2-1 Apply the counting principle with

$E_1 = \{1, \dots, 9\}$ and $E_i = \{0, 1, \dots, 9\}$ for $i = 2, \dots, 6$

where the element of E_i is the i-th digit in the number

$\Rightarrow$ # six-digit numbers $= 9 \cdot 10^5$

Analogously:

six-digit numbers without $5 = 8 \cdot 9^5$

$\Rightarrow$ # six-digit numbers containing $5 = 9 \cdot 10^5 - 8 \cdot 9^5$

2.2-3 Yes. # initials with 3 letters $= 26^3 = 17{,}576 < 20{,}000$

Since there are more people than possible choices of initials, at least two must have the same.

2.2-5 # combinations in 23 throws of coin $= 2^{23}$

A = all heads or all tails

$P(A) = \frac{|A|}{|S|} = \frac{2}{2^{23}} = \underline{\underline{2^{-22}}} = 2.4 \cdot 10^{-7}$

2.2-8 # outfit combinations $= 12 \cdot 8 \cdot 8 \cdot 4$

blue outfit combinations $= 4 \cdot 3 \cdot 2 \cdot 2$

$\Rightarrow P(\text{all blue outfit}) = \frac{4 \cdot 3 \cdot 2 \cdot 2}{12 \cdot 8 \cdot 8 \cdot 4} = \underline{\underline{\frac{1}{64}}} = 0.015625$

2.2-9 # combinations of answers $= 4^{15}$

$\Rightarrow P(\text{all right in random choice}) = \frac{1}{4^{15}} = 9.3 \; 10^{-10}$

2.2-11 # combinations of three letters followed by two numbers

$= 26^3 \cdot 10^2 = 1{,}757{,}600 > 800{,}000$

Yes, there are enough combinations to label the books.

2.2-17

card combinations with 4 draws $= 52^4$

card combinations with 4 draws and no king $= 48^4$

$\Rightarrow$ # card combinations with 4 draws, at least one king $= 52^4 - 48^4$

$P(\text{at least one king}) = \frac{52^4 - 48^4}{52^4} = 1 - \left(\frac{14}{15}\right)^4 = 0.274$

2.3-2 $\boxed{a}\underbrace{\square\square\square}_{\{b,d,e\}}\boxed{c}$ # permutations starting with a, ending with c

= # permutations of $\{b,d,e\} = 3! = \underline{\underline{6}}$

2.3-6 This is equivalent to asking

ordered pairs out of 6 elements $= 6 \cdot 5 = \underline{\underline{30}}$

2.3-10 This is equivalent to finding the number of words that can be formed from the letters AAAMMRRRIII

Permutations for n letters = n!

$\Rightarrow$ there are 11! permutations of the letters but we have to take into consideration that within those permutation of the same letters form the same word

$$\Rightarrow \# \text{ lists} = \frac{11!}{3!\,2!\,3!\,3!} = 92{,}400$$

[Alternatively, think of the 11 places in the list, first draw three numbers (the order being unimportant) that will be occupied by A's, then from the remaining places two occupied by M's...

$$\Rightarrow \# \text{ lists} = \binom{11}{3}\cdot\binom{8}{2}\cdot\binom{6}{3}\cdot\binom{3}{3} = \frac{11!}{3!2!3!3!} = 92{,}400.$$]

2.4-1 $\binom{20}{6} = \frac{20!}{6!\,14!} = 38{,}760$

2.4-3 $\binom{20}{6}\cdot\binom{25}{6} = \frac{20!\,25!}{6!\,14!\,6!\,19!} = 6{,}864{,}396{,}000$

2.4-5 $\# \text{Samples} = \binom{N}{n} = \frac{N!}{n!\,(N-n)!}$

Samples including a particular individual

$$= \binom{N-1}{n-1} = \frac{(N-1)!}{(n-1)!\,\underbrace{(N-n)}_{(N-1)-(n-1)}!}$$

(first separating the individual and then choosing the rest of the sample from the rest of the population)

$\Rightarrow$ P(particular ind. in sample)

$$= \frac{(N-1)!}{(n-1)!\cancel{(N-n)!}} \cdot \frac{\overset{n\cdot(n-1)!}{\cancel{n!}}\cancel{(N-n)!}}{\underbrace{N!}_{N\cdot(N-1)!}} = \underline{\underline{\frac{n}{N}}}$$

(Of course, one could have argued simpler: n in sample, N in population $\Rightarrow$ P(ind. in sample) $= \frac{n}{N}$.

2.4-11 $(2x-4y)^7 = \sum_{i=0}^{7} \binom{7}{i} (2x)^{7-i} (-4y)^i$ by binomial formula

$\Rightarrow \binom{7}{4}$ is the coefficient for $(2x)^3(-4y)^4$

$\Rightarrow \binom{7}{4} \cdot 2^3 \cdot (-4)^4$ is the coefficient for x^3y^4

$= \underline{\underline{71,680}}$

2.4-13 (a) Five outcomes out of ten are heads

$\Rightarrow \# = \binom{10}{5}$

$$\Rightarrow P(\text{five heads}) = \frac{\binom{10}{5}}{\underbrace{2^{10}}_{\#\text{ all possible outcomes}}} = \underline{\underline{0.246}}$$

(b)

$$P(\text{at least five heads}) = \sum_{i=5}^{10} P(\text{exactly } i \text{ heads}) = \sum_{i=5}^{10} \frac{\binom{10}{i}}{2^{10}} = \underline{\underline{0.623}}$$

2.4-15 (a) Choosing 2 out of 6 professors and 4 out of 28 non-professors when there are 6 out of 34 options:

$$\frac{\binom{6}{2} \cdot \binom{28}{4}}{\binom{34}{6}} = \underline{\underline{0.228}}$$

(b) Analogously: Summing up # all professors, associates...

$$\frac{\binom{6}{6} + \binom{6}{6} + \binom{10}{6} + \binom{12}{6}}{\binom{34}{6}} = \frac{1+1+210+924}{1,344,904} \approx \underline{\underline{8.4 \cdot 10^{-4}}}$$

2.4-21 Assuming that the contents of the box are divided evenly:

$$P(\text{6 bolts and 6 nuts for both}) = \frac{\binom{12}{6} \cdot \binom{12}{6}}{\binom{24}{12}} = 0.3157$$

2.4-39 The expression equals

$\sum_{k=0}^{n} \binom{n}{k} 1^{(n-k)} (-1)^k = (1-1)^n = 0$ by the binom. Theorem.