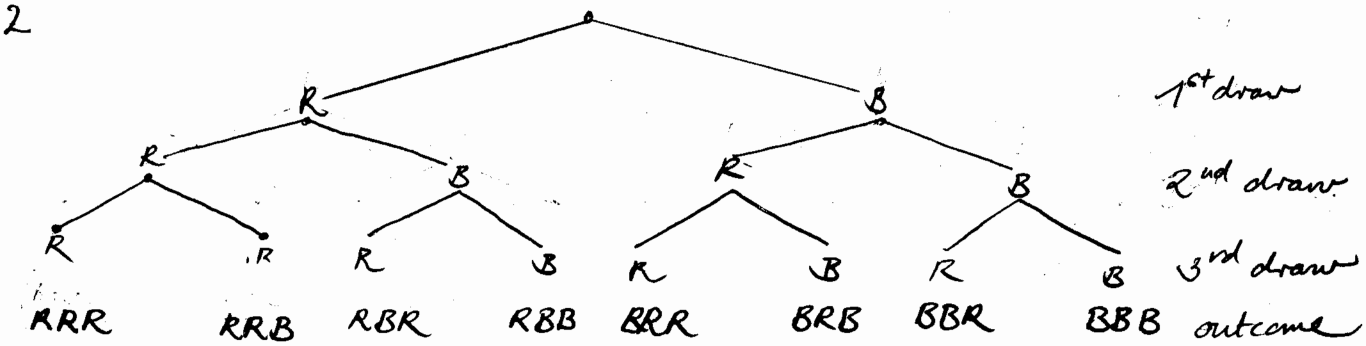
Solutions to Homework MATH 350

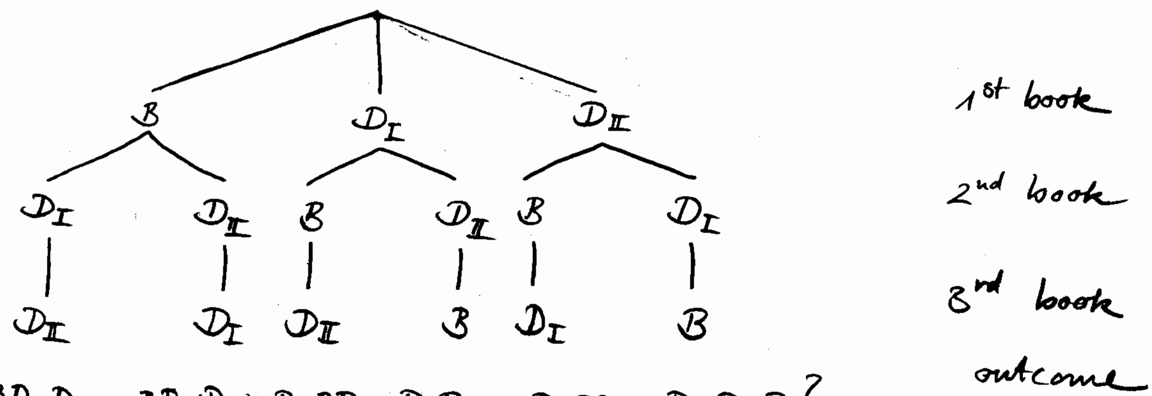
Chapter 1:

1.2-2



$\Rightarrow$ Sample space $S = \{RRR, RRB, RBR, RBB, BRR, BRB, BBR, BBB\}$

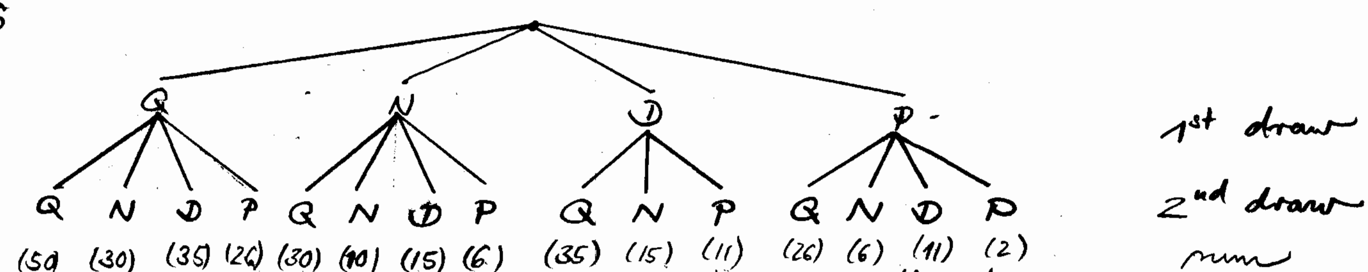
1.2-4



$\Rightarrow S = \{BD_ID_{II}, BD_{II}D_I, D_IBD_{II}, D_ID_{II}B, D_{II}BD_I, D_{II}D_IB\}$

$A :=$ event of dictionaries in increasing order side by side
$= \{BD_ID_{II}, D_ID_{II}B\}$

1.2-6



Note that in this experiment the order of the draws does not matter meaning that, i.e. $QN = NQ$

$\Rightarrow S = \{QQ, QN, QD, QP, NN, ND, NP, DP, PP\}$

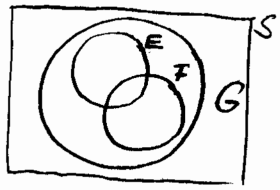
$A :=$ event of drawing 26 cents $= \{QP\}$

$B :=$ event of drawing more than less than 25 cents $= \{NN, DP, DN, ND, PD\}$

$C :=$ event of drawing 29 cents $= \emptyset$

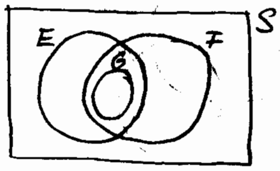
1.2-8 E, F, G events

$E \cup F \cup G = G$



G contains E and F
"If E or F occur then G occurs"

$EFG (= E \cap F \cap G) = G$



G is contained in the intersection of E and F
"If G occurs, then E and F occur"

1.4-1 No.

The sample space $S = \{(i,j) : 1 \leq i \leq 6,\ 1 \leq j \leq 6\}$

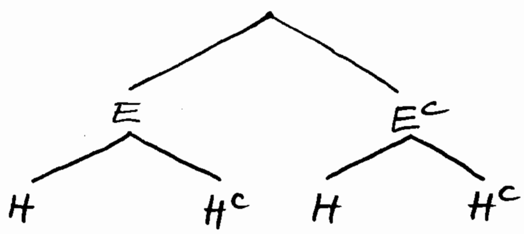
$A :=$ event that sum $i+j = 11 = \{(5,6),(6,5)\}$

$B :=$ event that sum $i+j = 12 = \{(6,6)\}$

Each throw is equally likely, it has the same probability $\frac{1}{36}$ since there are 36 possible combinations of throws.

$\Rightarrow P(A) = \frac{2}{36} \neq P(B) = \frac{1}{36}$

1.4-3



E: earthquake
H: hurricane

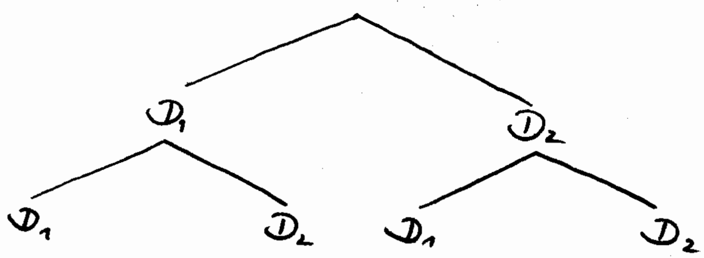
$\Rightarrow S = \{EH, EH^c, E^cH, E^cH^c\}$

We know that
$$P(EH) + P(EH^c) = 0.015$$
$$P(EH) + P(E^cH) = 0.025$$
$$P(EH) = 0.0073$$

$$\Rightarrow P(EH^c) = 0.015 - 0.0073 = 0.0077$$
$$P(E^cH) = 0.025 - 0.0073 = 0.0177$$

$$P(E^cH^c) = 1 - P(EH \dot\cup E^cH \dot\cup EH^c)$$
$$= 1 - (P(EH) + P(E^cH) + P(EH^c))$$
$$= 1 - 0.0073 - 0.0177 - 0.0077 = \underline{\underline{0.9673}}$$

1.4-7 No:



1^{st} autopsy

2^{nd} autopsy

$\Rightarrow S = \{D_1D_1, D_1D_2, D_2D_1, D_2D_2\}$

Assuming that both doctors are equally likely to perform a given autopsy we obtain a uniform distribution on S:

$P(D_1D_1) = P(D_1D_2) = P(D_2D_1) = P(D_2D_2) = \frac{1}{4}$

$\Rightarrow P(\underbrace{D_1D_1 \,\dot\cup\, D_2D_2}_{\text{"same doctor performs the two autopsies"}}) = P(D_1D_1) + P(D_2D_2) = \frac{1}{2}$

1.4-9

Let $p = P$(event that randomly selected patient suffers from schizophrenia)

$\Rightarrow \frac{1}{3}p = P$(psychoneurosis)

$\frac{1}{2}p = P$(addiction)

$\frac{1}{10}p = P$(psychosis)

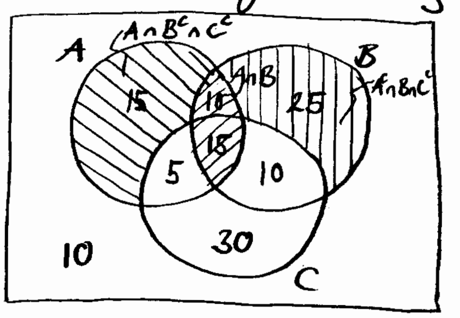
(Assuming that each patient is afflicted by exactly one of the diseases)

$\Rightarrow p + \frac{1}{3}p + \frac{1}{2}p + \frac{1}{10}p = 1$

$\frac{58}{30}p = 1$

$\underline{\underline{p = \frac{30}{58} = \frac{15}{29}}}$

1.4-19 Venn diagram:



We know that

1. $|S| = 120$
2. $|A \cap C| = 20$
3. $|A \cap B \cap C^c| = 10$
4. $|A \cap B \cap C| = 15$
5. $|A^c \cap B^c \cap C| = 30$
6. $|B \cap C^c| = 35$
7. $|B \cap C| = 25$
8. $|A^c \cap B^c \cap C^c| = 10$

$\Rightarrow$ 3.+4. $\Rightarrow |A \cap B| = 25$

2.+4. $\Rightarrow |A \cap B^c \cap C| = 5$

4.+7. $\Rightarrow |A^c \cap B \cap C| = 10$

6.+7. $\Rightarrow |B| = 60$

$\Rightarrow |A^c \cap B \cap C^c| = 25$

1 + above

$\Rightarrow |A \cap B^c \cap C^c| = 15$

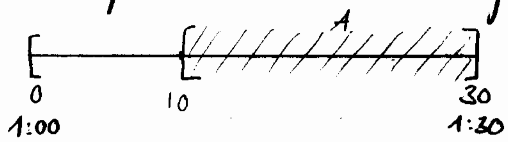
$\Rightarrow P(A \cap B^c \cap C^c) = \frac{|A \cap B^c \cap C^c|}{|S|} = \frac{15}{120} = \underline{\underline{\frac{1}{8}}}$

$P(A^c \cap B \cap C^c) = \frac{|A^c \cap B \cap C^c|}{|S|} = \frac{25}{120} = \underline{\underline{\frac{5}{24}}}$

$P(A \cap B) = \frac{|A \cap B|}{|S|} = \frac{25}{120} = \underline{\underline{\frac{5}{24}}}$

1.7-1 $S = [0, 30]$

A = person has to wait for more than 10 minutes $= [10, 30]$



$$\Rightarrow P(A) = \frac{|[10,30]|}{|[0,30]|} = \frac{20}{30} = \underline{\underline{\frac{2}{3}}}$$

1.7-5 $S = (0, 2000)$

A = number is integer $= \{1, 2, \dots, 1998, 1999\}$

$$P(A) = \sum_{i=1}^{1999} P(\{i\}) = \sum_{i=1}^{1999} \frac{|\{i\}|}{|(0,2000)|} = \sum_{i=1}^{1999} \frac{0}{2000} = 0.$$

the interval length or area of points is 0!

1.7-10 The set of rational numbers is countable.

Thus $\bar{Q} = Q \cap (0,1) = \{r_1, r_2, \dots\}$.

= number is rational

$$\Rightarrow P(\bar{Q}) = \sum_{i=1}^{\infty} P(\{r_i\}) = \sum_{i=1}^{\infty} 0 = \underline{\underline{0}}$$

$\Rightarrow$ $\bar{Q}^c$ = number is irrational

$$P(\bar{Q}^c) = 1 - P(\bar{Q}) = 1 - 0 = \underline{\underline{1}}$$