\( a) \quad p = 0.6, \quad q = 0.4 \\
Pr(4 \text{ success in 6 trials}) = C(6, 4)(0.6)^4(0.4)^2 = 0.31104 \\
Pr(2 \text{ failures in 6 trials}) = C(6, 2)(0.4)^2(0.6)^4 = 0.2304

\( b) \quad p = 0.4, q = 0.6 \\
Pr(3 \text{ success in 5 trials}) = C(5, 3)(0.4)^3(0.6)^2 = 0.09883

\( c) \quad \text{At least 3 successes in 6 trials, } p = 0.2 \\
1 - Pr(0) - Pr(1) - Pr(2) = 1 - C(6, 0)(0.2)^0(0.8)^6 - C(6, 1)(0.2)^1(0.8)^5 - C(6, 2)(0.2)^2(0.8)^4 = 0.09883

\( d) \quad \text{At least 3 failures in 5 trials, } p = 0.8 \\
Pr(3 +) + Pr(4 +) + Pr(5 +) = C(5, 3)(0.8)^3 + C(5, 4)(0.8)^4 + C(5, 5)(0.8)^5 = 0.05792

\( e) \quad \text{Exactly 2 heads or exactly 2 tails, } p(\text{H}) = 0.4. \quad \text{Flip 5 times} \\
Pr(2 \text{H}) + Pr(2 \text{T}) = C(5, 2)(0.4)^2(0.6)^3 + C(5, 2)(0.6)^2(0.4)^3 = 0.576
13. \( p = 0.6 \) (each is \( R \)) \( \Rightarrow \) \( m = 5 \) days,

\[ \bar{m} = 0.4 \]

a) \( Pr(1) = C(5,1)(0.6)^1(0.4)^4 = \underline{0.0768} \)

b) \( Pr(5) = C(5,5)(0.6)^5(0.4)^0 = \underline{0.07776} \)

c) \( Pr(\text{at most 1}) = Pr(0) + Pr(1) \)

\[ = C(5,0)(0.6)^0(0.4)^5 + Pr(1) \text{ (known from part (a))} \]

\[ = \underline{0.08704} \]

15. \( p = 0.5 \) 20 trials

\( Pr(15,5) = C(20,5)(0.5)^5(0.5)^{15} = C(20,5)(0.5)^20 \approx 0.00018 \)

29. \( p = \frac{1}{2} \) (demutant = 5) 10 trials

\( Pr(\text{at least 7}) = Pr(7) + Pr(8) + Pr(9) + Pr(10) \)

\[ = C(10,7)(\frac{1}{2})^7(\frac{1}{2})^3 + C(10,8)(\frac{1}{2})^8(\frac{1}{2})^2 + \cdots \]

\[ = (\frac{1}{2})^{10}(C(10,7) + C(10,8) + C(10,9) + C(10,10)) \]

\[ = 120 + 45 + 10 + 1 \]

\[ = \frac{176}{1024} \text{ (check to know this.)} \]

\[ = \frac{0.171}{64} \]
(3) 10 questions, 5 choices. \( p = \frac{1}{5} \).

\[
\Pr(2 \text{ students answer 4 correctly by guessing}) = \left[\Pr(4)\right]^2 = \left[ \binom{10}{4} \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right)^6 \right]^2 \approx 0.00746
\]

\( \frac{3.5}{4.1} \)

---

1. \( \Pr(H) = .4 = p \)

Flip 3 times

\( X = H + 2(T) \)

a) \( \left( X | H, T, T \right) \sim \mathcal{N}(0.3, (1, 2), (2, 1), (1, 0)) \)

So \( \{X\} = \{6, 5, 4, 3\} \)

but write in order: \( X = 3, 4, 5, 6 \)

b) \( \Pr(X = 3) = \binom{3}{3} (0.4)^3 (0.6)^0 = 0.064 \)

\( \Pr(X = 4) = \binom{3}{2} (0.4)^2 (0.6)^1 = 0.288 \)

\( \Pr(X = 5) = \binom{3}{1} (0.4)^1 (0.6)^2 = 0.432 \)

\( \Pr(X = 6) = \binom{3}{0} (0.4)^0 (0.6)^3 = 0.216 \)

<table>
<thead>
<tr>
<th>( X )</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.064</td>
</tr>
<tr>
<td>4</td>
<td>0.288</td>
</tr>
<tr>
<td>5</td>
<td>0.432</td>
</tr>
<tr>
<td>6</td>
<td>0.216</td>
</tr>
</tbody>
</table>
4 females
2 males
Select two

\(X = \text{# of females}\)

\[
\begin{array}{c|c}
\text{Probability} \\
0 & \frac{1}{15} \\
1 & \frac{8}{15} \\
2 & \frac{2}{5}
\end{array}
\]

\[
\begin{align*}
\Pr(X = 0) &= \frac{\binom{2}{2}}{\binom{4}{2}} = \frac{1}{15} \\
\Pr(X = 1) &= \frac{\binom{2}{1} \cdot \binom{2}{1}}{\binom{4}{2}} = \frac{8}{15} \\
\Pr(X = 2) &= \frac{\binom{4}{2}}{\binom{6}{2}} = \frac{6}{15} = \frac{2}{5}
\end{align*}
\] (Note: I am not using the Bernoulli equation because choices are dependent.)
7. 5 women, 3 men. Choose 3. \( X = \# \) women.

\[
\Pr(X = 2) = \frac{C(5, 2) \cdot C(3, 1)}{C(8, 3)} = \frac{10}{28}
\]

8. One 1, two 3s, one 5. Choose two.

\( X = \text{sum}. \)

\[
\{\text{sum} 1, \text{sum} 2\} = \{1, 5\}, \{1, 5\}, \{3, 3\}, \{3, 5\}
\]

So \( \{X\} = \{4, 6, 8\} \)

\[
\Pr(X = 4) = \Pr(1, 3) = \frac{C(1, 1) \cdot C(3, 1)}{C(4, 2)} = \frac{1}{3}
\]

\[
\Pr(X = 6) = \Pr(1, 5) + \Pr(3, 3) = \frac{C(1, 1) \cdot C(1, 1)}{C(4, 2)} + \frac{C(3, 2)}{C(4, 2)} = \frac{1}{3}
\]

\[
\Pr(X = 8) = \Pr(3, 5) = \frac{C(3, 2)}{C(4, 2)} = \frac{1}{3}
\]

<table>
<thead>
<tr>
<th>( X )</th>
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</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>6</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>8</td>
<td>( \frac{1}{3} )</td>
</tr>
</tbody>
</table>
2 red, 3 green. Choose two, replace only if first is green.

\[ X := \# \text{red} \]

\[ \Pr(X=0) = \Pr(G,G) = \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25} \]

\[ \Pr(X=1) = \Pr(R,G) + \Pr(G,R) = \frac{2}{5} \cdot \frac{3}{5} + \frac{3}{5} \cdot \frac{2}{5} = \frac{23}{50} \]

\[ \Pr(X=2) = \Pr(R,R) = \frac{2}{5} \cdot \frac{2}{5} = \frac{1}{10} \]

<table>
<thead>
<tr>
<th>( X )</th>
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</tr>
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<tbody>
<tr>
<td>0</td>
<td>( \frac{9}{25} ) = .36</td>
</tr>
<tr>
<td>1</td>
<td>( \frac{23}{50} ) = .46</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{1}{10} ) = .10</td>
</tr>
</tbody>
</table>

Roll fair die until sum exceeds 3. \( X := \# \text{rolls} \).

Note: \( X \leq 4 \), since 4 rolls at most.

\[ \Pr(X=1) = \Pr(1, 5 \text{ or } 6) = \frac{1}{6} \]

\[ \Pr(X=2) = \Pr(1, \text{ any } 3, 4, 5 \text{ or } 6) + \Pr(2, \text{ any } 3, 4, 5 \text{ or } 6) + \Pr(3, \text{ any } \text{ anything}) \]

\[ = \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} = .1667 \]

\[ \Pr(X=3) = \Pr(1, \text{ any } 2, 3, \ldots, 6) + \Pr(2, \text{ any } \text{ anything}) + \Pr(3, \text{ anything}) \]

\[ = \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} = .0787 \]

\[ \Pr(X=4) = \Pr(1, 1, \text{ anything}) \]

\[ = \left( \frac{1}{6} \right)^3 \cdot \frac{1}{6} = .0046 \]

<table>
<thead>
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<tbody>
<tr>
<td>1</td>
<td>.1667</td>
</tr>
<tr>
<td>2</td>
<td>.0787</td>
</tr>
<tr>
<td>3</td>
<td>.0046</td>
</tr>
<tr>
<td>4</td>
<td>.0046</td>
</tr>
</tbody>
</table>
Two coins: one fair, one with \( p(H) = \frac{2}{3} \).

Select a coin, flip twice.

\[ X = \#H \]

\[
\begin{align*}
\Pr(X = 0) &= \frac{1}{2} \left( \frac{1}{2} \right)^2 + \frac{1}{2} \left( \frac{1}{3} \right)^2 = \frac{13}{72} \\
\Pr(X = 1) &= \frac{1}{2} \binom{2}{1} \left( \frac{1}{2} \right) \left( \frac{1}{3} \right)^1 + \frac{1}{2} \binom{2}{1} \left( \frac{2}{3} \right) \left( \frac{1}{3} \right)^1 = \frac{34}{72} \\
\Pr(X = 2) &= \frac{1}{2} \left( \frac{1}{2} \right)^2 + \frac{1}{2} \left( \frac{2}{3} \right)^2 = \frac{25}{72}
\end{align*}
\]

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<tr>
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</tr>
<tr>
<td>1</td>
<td>( \frac{34}{72} )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{25}{72} )</td>
</tr>
</tbody>
</table>
1) \( E[X] = (1)(.5) + (2)(.5) + (8)(.3) = 3.5 \)

2) \( E[X] = \frac{1}{21}(1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2) = \frac{91}{21} \)

3) 3 Nickels, 2 Dimes, 4 Quarters. Select one.
   \( E[X] = (\frac{3}{5})(\frac{2}{5}) + (\frac{1}{10})(\frac{2}{5}) + (\frac{4}{25})(\frac{4}{5}) = 15 \frac{4}{5} \)

4) 4 Blue, 3 Green. Select 4.
   \( X = \# \text{Blue} - \# \text{Green} \)
   \( E[X] = (-2) \Pr(1, 4, 3) \cdot (-2) \cdot \frac{C(4,1) \cdot C(3,3)}{C(7,4)} + (2) \frac{C(4,3) \cdot C(3,1)}{C(7,4)} + (4) \frac{C(4,4)}{C(7,4)} = \frac{4}{7} \)

5) Coin: \( Pr(H) = \frac{2}{5} \). Flip 10 times.
1+ \quad \text{Pr}(H) = .6, \text{ flip until H or 4 flips, } X = \# \text{flips.}

\begin{align*}
\text{Pr}(X = 1) &= \text{Pr}(H) = .6 \\
\text{Pr}(X = 2) &= \text{Pr}(TH) = .4 \times .6 = .24 \\
\text{Pr}(X = 3) &= \text{Pr}(TTH) = .4^2 \times .6 = .096 \\
\text{Pr}(X = 4) &= \text{Pr}(TTTH) = .4^3 \times .6 + .4^4 = .064
\end{align*}

Then \quad E(X) = (1)(.6) + (2)(.24) + (3)(.096) + (4)(.064) = 1.624