1. \( \Pr(H) = 0.2 \), \( \Pr(T) = 0.8 \)  
Flip twice.

\[
\begin{align*}
\Pr(HH) &= 0.04 \\
\Pr(HT) &= 0.16 \\
\Pr(TH) &= 0.16 \\
\Pr(TT) &= 0.64
\end{align*}
\]

(multiply along branches)

2. \( \Pr(\#) = \frac{1}{8} \), \( \Pr(T) = \frac{2}{3} \)

9 cards: 4 aces, 3 kings, 2 queens. Choose 1.

\[
\begin{align*}
\Pr(\#A) &= \frac{4}{27} \\
\Pr(\#K) &= \frac{2}{27} \\
\Pr(\#Q) &= \frac{2}{27}
\end{align*}
\]

3. \( \Pr(HA) = \frac{4}{27} \), \( \Pr(HK) = \frac{2}{27} \), \( \Pr(HQ) = \frac{2}{27} \)

\[
\begin{align*}
\Pr(TA) &= \frac{8}{27} \\
\Pr(TK) &= \frac{2}{27} \\
\Pr(TQ) &= \frac{2}{27}
\end{align*}
\]
7. 9 Apples: 5 Red, 4 Yellow.
Pick Two: No replacement.

The problem is delicious.

\[ \Pr(RR) = \frac{20}{36} = \frac{5}{9} \]
\[ \Pr(RY) = \frac{20}{36} = \frac{5}{9} \]
\[ \Pr(YR) = \frac{10}{36} = \frac{5}{18} \]
\[ \Pr(YY) = \frac{12}{36} = \frac{1}{6} \]

a) \( \Pr(\text{second is red}) = \Pr(RR) + \Pr(YR) = \frac{5}{9} + \frac{5}{18} = \frac{10}{18} = \frac{5}{9} \)

b) \( \Pr(\text{at least one red}) = 1 - \Pr(\text{no red}) = 1 - \Pr(YY) = 1 - \frac{1}{6} = \frac{5}{6} \)

Note: These problems can be done with combinations, but be careful with part (a) because order matters.

13. 7 balls: 4 Red, 3 Blue. Choose Two, no replacement.

\[ \frac{3}{4} \]
\[ \frac{1}{4} \]
\[ \frac{3}{4} \]

\[ \frac{1}{4} \]
\[ \frac{3}{4} \]
\[ \frac{1}{4} \]

a) \( \Pr(\text{exactly 1 Blue}) = \Pr(\text{RB}) + \Pr(\text{BR}) = \frac{3}{7} + \frac{3}{7} = \frac{6}{7} \)

b) \( \Pr(\text{2 Blue | at least one blue}) = \frac{\Pr(\text{RB}) + \Pr(\text{BB})}{1 - \Pr(\text{RB})} = \frac{3}{7} + \frac{3}{7} = \frac{6}{7} \)
15. Indianapolis to Metroburg:

The real question is: why are we flying from a real city to a fake one?

\[ Pr(\text{on time}) = Pr(\text{Urban on time}) + Pr(\text{Bi-Coastal on time}) + Pr(\text{DVD on time}) \]

\[ = (.5)(.8) + (.2)(.6) + (.3)(.4) \]

\[ = .4 + .12 + .12 = .64 \]

19. \( Pr(H) = .2 \). Flip until 3 total heads or 5 flips.

\[ Pr(\text{exactly one head}) = \frac{\text{Pr}((HTTTT) + \text{Pr}(TTTTT)) + \text{Pr}(TTTTT) + \text{Pr}(TTTTT)}{\text{Pr}(TTTTT)} \]

\[ = \frac{(2\cdot 8\cdot 6\cdot 4\cdot 2) + (8\cdot 6\cdot 4\cdot 2) + (8\cdot 6\cdot 4\cdot 2) + (8\cdot 6\cdot 4\cdot 2) + (8\cdot 6\cdot 4\cdot 2) + (8\cdot 6\cdot 4\cdot 2)}{8\cdot 6\cdot 4\cdot 2} = .4096 \]

Note: we do not need a tree for this.
\[ \Pr(\text{profit exactly 2 of 3 years}) = \binom{3}{2}(0.6)(0.4) + \binom{3}{2}(0.4)(0.6) + (0.8)(0.4)(0.6) \]
\[ = 0.048 + 0.032 + 0.192 = 0.272 \]

29. Rolls 1 through 7. Replace even only. Pick two. Find \( \Pr(\text{one is 5}). \)

\[ \Pr(\text{one is 5}) = \frac{1}{6} + \frac{3}{6} \cdot \frac{1}{2} + \frac{3}{6} \cdot \frac{1}{4} = \frac{76 + 72 + 63}{216} = \frac{211}{216} = \frac{3}{3} \cdot \frac{27}{216} = \frac{3}{27} = 0.27 \]
1. \[ \text{Pr}[B] = \frac{1}{2} \]
2. \[ \text{Pr}(B) = \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3} = \frac{2}{6} + \frac{1}{6} = \frac{2}{3} \]
3. \[ \text{Pr}(B) = \frac{2}{3} \]
4. \[ \text{Pr}(B \mid A) = \frac{\text{Pr}(B \cap A)}{\text{Pr}(A)} = \frac{\frac{1}{3} \cdot \frac{2}{3}}{\frac{1}{3}} = \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9} \]

3.4

A: 2 red, 2 green
B: 1 red, 2 green
Choose a box, then a ball.
Pr(B \mid G) = ?

Method one: no tree
\[ \text{Pr}(G \mid A) = \frac{1}{2} \]
\[ \text{Pr}(G \mid B) = \frac{2}{3} \]
\[ \text{Pr}(B \mid G) = \frac{\text{Pr}(G \mid B) \text{Pr}(B)}{\text{Pr}(G \mid A) \text{Pr}(A) + \text{Pr}(G \mid B) \text{Pr}(B)} = \frac{\frac{2}{3} \cdot \frac{1}{2}}{\frac{1}{2} + \frac{2}{3} \cdot \frac{1}{2}} = \frac{3}{4} \]

Method two: tree
\[ \text{Pr}(B \mid G) = \frac{\text{Pr}(B \cap G)}{\text{Pr}(G)} = \frac{\frac{1}{2} \cdot \frac{3}{4}}{\frac{1}{2} \cdot \frac{3}{4} + \frac{1}{2} \cdot \frac{3}{4}} = \frac{4}{7} \]
5) See Fig. 3.18, pg 122

\[ a) \Pr(C \text{ and } B | Y) = \frac{\Pr(C \text{ and } B \text{ and } Y)}{\Pr(Y)} = \frac{\frac{2}{5} \cdot \frac{1}{3} \cdot \frac{2}{5}}{\frac{2}{5}} = \frac{1}{5} \]

\[ b) \Pr(Y | (B \text{ and } B)) = \frac{\Pr(Y \text{ and } B \text{ and } B)}{\Pr(B \text{ and } B)} = \frac{\frac{2}{5} \cdot \frac{1}{3} \cdot \frac{2}{5}}{\frac{2}{5} \cdot \frac{4}{5} + \frac{3}{5} \cdot \frac{1}{5} \cdot \frac{2}{5}} = \frac{1}{5} \]

6) Wells: 2 red, 1 green, 5 blue. Take two, no replacement.

Find \( \Pr(\text{first blue} | \text{second red}) \).

\[ \Pr(\text{second red} | \text{first blue}) = \frac{2}{5} \]

\[ \Pr(\text{second red} | \text{first green}) = \frac{1}{5} \]

\[ \Pr(\text{second red} | \text{first red}) = \frac{1}{5} \]

\[ \Pr(\text{first blue} | \text{second red}) = \frac{\Pr(\text{2nd red} | 1^{st} \text{ blue}) \Pr(1^{st} \text{ blue})}{\Pr(\text{2nd red} | 1^{st} \text{ blue}) \Pr(1^{st} \text{ blue}) + \Pr(\text{2nd red} | 1^{st} \text{ green}) \Pr(1^{st} \text{ green}) + \Pr(\text{2nd red} | 1^{st} \text{ red}) \Pr(1^{st} \text{ red})} = \frac{\frac{2}{5} \cdot \frac{3}{6}}{\frac{2}{5} \cdot \frac{3}{6} + \frac{3}{5} \cdot \frac{1}{6} + \frac{1}{5} \cdot \frac{2}{6}} = \frac{6}{10} = \frac{3}{5} \]
(11) $\text{F} =$ fair coin
$\text{C} =$ cheater coin ($pr(H) = .6$)

Select a coin, flip twice. Find $pr(\text{F} | \text{HH})$.

$pr(\text{HH} | \text{F}) = .25$
$pr(\text{HH} | \text{C}) = .6^2 = .36$

$$pr(\text{F} | \text{HH}) = \frac{pr(\text{HH} | \text{F}) \cdot pr(\text{F})}{pr(\text{HH} | \text{F}) \cdot pr(\text{F}) + pr(\text{HH} | \text{C}) \cdot pr(\text{C})} = \frac{.25 \cdot .5}{.25 \cdot .5 + .36 \cdot .5} = \frac{25}{61}$$

(21) $pr(\text{K}) = .2$  \hspace{1cm} (L = signal toss)
$pr(\text{M}) = .4$
$pr(\text{N}) = .4$

Find $pr(\text{N} | \text{L})$.

$pr(\text{L} | \text{K}) = .1$
$pr(\text{L} | \text{N}) = .1$
$pr(\text{L} | \text{M}) = .05$

$$pr(\text{N} | \text{L}) = \frac{pr(\text{L} | \text{N}) \cdot pr(\text{N})}{pr(\text{L} | \text{N}) \cdot pr(\text{N}) + pr(\text{L} | \text{M}) \cdot pr(\text{M}) + pr(\text{L} | \text{K}) \cdot pr(\text{K})}$$

$$= \frac{(0.1)(0.4)}{(0.1)(0.4) + (0.05)(0.4) + (0.1)(0.2)} = \frac{0.5}{0.9}$$
A: 2 red, 1 green
B: 3 red, 2 green

Choose a box, then two balls. Find \( P(B|\text{ same color}) \).

\[
\begin{align*}
\text{Pr(same color | B)} &= \frac{\frac{3}{5} \cdot \frac{2}{4}}{\frac{2}{5}} + \frac{\frac{2}{5} \cdot \frac{1}{4}}{\frac{2}{5}} = \frac{2}{5} \\
\text{Pr(same color | A)} &= \frac{\frac{1}{3} \cdot \frac{1}{2}}{\frac{1}{3}} = \frac{1}{2} \\
\text{Pr (B | same color)} &= \frac{\text{Pr (same color | B) Pr (B)} + \text{Pr (same color | A) Pr (A)}}{\text{Pr (same color | B) Pr (B)} + \text{Pr (same color | A) Pr (A)}} \\
&= \frac{\frac{2}{5} \cdot \frac{1}{2}}{\frac{2}{5} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2}} = \frac{2}{5} \cdot \frac{1}{2} = \frac{6}{11}
\end{align*}
\]

(23)

1R, 1B, 1G.

Red ball: stop.

All blue: replace, pull again.

All green: don’t replace, pull again.

Find:

\( \text{Pr(1st blue | 2nd blue)} \)

\[
\begin{align*}
\text{Pr(2nd blue | 1st blue)} &= \frac{1}{3} \\
\text{Pr(2nd blue | 1st red)} &= 0 \\
\text{Pr(2nd blue | 1st green)} &= \frac{1}{2} \\
\text{Pr (1st B | 2nd B)} &= \frac{\text{Pr(2nd B | 1st B) Pr(1st B)}}{\text{Pr(2nd B | 1st B) Pr(1st B) + Pr(2nd B | 1st R) Pr(1st R) + Pr(2nd B | 1st G) Pr(1st G)}} \\
&= \frac{\frac{1}{3} \cdot \frac{1}{3}}{\frac{1}{3} \cdot \frac{1}{3} + 0 \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3}} = \frac{1}{5}
\end{align*}
\]