Exam 3
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Math 242 Section 450
Instructor: Patrick C. Rowe
Office: 323 Ewing Hall

Print Name: Answer Key

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By signing below, I acknowledge that I have read the above and that I have neither given nor received assistance on this examination.

Sign Name: Answer Key

All work must be shown to receive credit. Try to do all your work on this paper and clearly indicate your final answer. If you need additional paper, ask for it. All paper used must be turned in with your exam.

No Calculators may be used. All cellular phones and/or electronic devices of any kind must be turned off and put away. CHECK YOUR PHONE NOW!! If your phone rings during the exam, you may be asked to leave.
1. For the following limits: if it exists, calculate the value, otherwise, show it does not exist.

a. \( \lim_{(x,y) \to (0,0)} (5x + 3xy + y + 1) = 1 \)

b. \( \lim_{(x,y) \to (0,0)} e^{xy} = 1 \)

c. \( \lim_{(x,y) \to (0,0)} \frac{x^2y^2}{x^2 + y^2} = \lim_{r \to 0} \frac{r^4 \sin^2 \theta \cos^2 \theta}{r^2} = \lim_{r \to 0} r^2 \sin^2 \theta \cos^2 \theta = 0 \)

d. \( \lim_{(x,y) \to (0,0)} \frac{xy}{x^2 + y^2} \)

Along \( y = x \)

\( \lim_{(x,y) \to (0,0)} \frac{x^2}{2x^2} = \lim_{x \to 0} \frac{x^2}{2x^2} = \frac{1}{2} \)

But along \( x = 0 \)

\( \lim_{(x,y) \to (0,0)} \frac{0}{y^2 + y^2} = 0 \)

\( \therefore \text{NE} \)
2. Find the indicated partial derivatives.

a. \( f(x, y) = \sqrt{x^2 + y^2} \). \( \frac{\partial f}{\partial y} \) ______________________ \\
\( \frac{f_y}{\sqrt{x^2 + y^2}} \)


b. \( f(x, y) = x^2 \sin(xy) \). \( \frac{\partial f}{\partial x} \) ______________________

\[ f_x = 2x \sin(xy) + xy \cos(xy) \]


c. \( f(x, y) = xyz \). \( \frac{\partial^2 f}{\partial y \partial x} \) and \( \frac{\partial^2 f}{\partial x \partial y} \) ______________________

\[ f_{xy} = f_{yx} = z \]

d. \( \tan(x + y) + \tan(y + z) = 1 \). \( \frac{\partial z}{\partial y} \) ______________________

\[ F_y = \sec^2(x + y) + \sec^2(y + z) \]

\[ F_z = \sec^2(y + z) \]

\[ \frac{\partial^2 z}{\partial y \partial z} = -\frac{F_y}{F_x} = \frac{- \left( \sec^2(x + y) + \sec^2(y + z) \right)}{\sec^2(y + z)} \]

e. \( w = y^3 - 3x^2y \). \( x = e^t, y = t^2 \). \( \frac{\partial w}{\partial x} \) and \( \frac{\partial w}{\partial t} \) ______________________

\[ \frac{\partial w}{\partial x} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial t} = -6xy \cdot e^t = -6e^t e^2 = -6e^{3t} \]

\[ \frac{\partial w}{\partial t} = \frac{\partial w}{\partial y} \frac{\partial y}{\partial t} = (3y^2 - 3x^2) \cdot 2t = 6t(e^t - e^{3t}) \]

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3
3. Find the directional derivative of \( f \) at \( P \) in the direction of \( \mathbf{v} \).

\[
f(x, y) = e^x \sin y, \quad P = (1, \frac{\pi}{2}), \quad \mathbf{v} = \langle -1, 0 \rangle
\]

\[
\nabla f = \langle e^x \sin y, e^x \cos y \rangle \Rightarrow \nabla f(1, \frac{\pi}{2}) = \langle e, 0 \rangle
\]

\[
D_{\mathbf{v}} f = \nabla f(1, \frac{\pi}{2}) \cdot \mathbf{v} = \langle e, 0 \rangle \cdot \langle -1, 0 \rangle = -e
\]

4. Find an equation for the tangent plane to the indicated surface at the indicated point.

\[
xz + yz + xy = 9, \quad P = (1, 1, 4)
\]

\[
\mathbf{f}(x, y, z) = xz + yz + xy
\]

\[
\nabla \mathbf{f} = \langle z + y, z + x, x + y \rangle
\]

\[
\nabla \mathbf{f}(1, 1, 4) = \langle 5, 5, 5 \rangle
\]

Tangent Plane

\[
5(x - 1) + 5(y - 1) + 5(z - 4) = 0
\]
5. Minimize \( f(x, y, z) = x^2 + y^2 + z^2 \) with constraint \( x + y + z^2 = 1 \) by using Lagrange multipliers.

\[
\begin{align*}
\nabla f &= \lambda \nabla g \\
2x &= \lambda & (1) \\
2y &= \lambda & (2) \\
2z &= 2z \lambda & (3) \\
x + y + z^2 &= 1 & (4)
\end{align*}
\]

(3) \( \Rightarrow \) \( 2z(1-\lambda) = 0 \) \( \Rightarrow \) \( z = 0 \) or \( \lambda = 1 \)

\( \lambda = 1 \)

(1) \( \Rightarrow \) \( x = \frac{1}{2} \) ; (3) \( \Rightarrow \) \( y = \frac{1}{2} \)

Sub (1) into (4) to get \( z = 0 \)

Together we get \( P_1 = (\frac{1}{2}, \frac{1}{2}, 0) \)

\( z = 0 \)

(1) \( \Rightarrow \) \( x = 1 - y \) put into (3) \( \Rightarrow 2(1-y) = \lambda \) (5)

(5) - (4) \( \Rightarrow 2 - 2y - y = 0 \) \( \Rightarrow 2 = 3y \) \( \Rightarrow y = \frac{1}{3} \) \( \Rightarrow x = \frac{1}{2} \)

We get the same pt (of course)

\( f(\frac{1}{2}, \frac{1}{3}, 0) = \frac{1}{4} + \frac{1}{9} = \frac{13}{36} \)