Exam 2
October 21, 2008

Math 243 Section 450
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Print Name: [Answer Key]

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By signing below, I acknowledge that I have read the above and that I have neither given nor received assistance on this examination.

Sign Name: [Answer Key]

All work must be shown to receive credit. Try to do all your work on this paper and clearly indicate your final answer. If you need additional paper, ask for it. All paper used must be turned in with your exam.

No Calculators may be used. All cellular phones and/or electronic devices of any kind must be turned off and put away. CHECK YOUR PHONE NOW!! If your phone rings during the exam, you may be asked to leave.

1
1. Complete the table by converting the coordinates to the indicated systems. Assume that \(-\pi < \theta \leq \pi\) for cylindrical and spherical coordinates.

<table>
<thead>
<tr>
<th>Cartesian</th>
<th>Cylindrical</th>
<th>Spherical</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\sqrt{3}, -1, 2))</td>
<td>((2, \frac{\sqrt{3}}{2}, 2))</td>
<td>((2\sqrt{3}, -\frac{\pi}{3}, \frac{\pi}{6}))</td>
</tr>
<tr>
<td>((\frac{\sqrt{6}}{2}, -\frac{\sqrt{2}}{2}, 1))</td>
<td>((3, \frac{\pi}{4}, 1))</td>
<td>((\sqrt{10}, -\frac{\pi}{4}, \cos^{-1} \left(\frac{1}{5}\right)))</td>
</tr>
<tr>
<td>((\frac{7}{2}, \frac{7}{2}, -\frac{15}{2}))</td>
<td>((\frac{7\sqrt{5}}{2}, \frac{\pi}{4}, \frac{\pi}{2}))</td>
<td>((\frac{7}{4}, \frac{3\pi}{4}))</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\rho &= 1 + 4 = 5 \\
\theta &= \tan^{-1} \left(\frac{3}{4}\right) \\
\phi &= \cos^{-1} \left(\frac{\sqrt{5}}{5}\right)
\end{align*}
\]

\[
\begin{align*}
x &= \sqrt{3} \cos \left(-\frac{\pi}{4}\right) = \frac{\sqrt{6}}{2} \\
y &= 3 \sin \left(\frac{\pi}{4}\right) = \frac{3\sqrt{2}}{2} \\
p &= 5 + 1 = 6 \\
\theta &= \cos^{-1} \left(\frac{1}{5}\right)
\end{align*}
\]

\[
\begin{align*}
x &= 7 \sin \left(\frac{\pi}{4}\right) \cos \left(\frac{\pi}{6}\right) = \frac{7\sqrt{2}}{2} \\
y &= 7 \sin \left(\frac{\pi}{4}\right) \sin \left(\frac{\pi}{6}\right) = \frac{7\sqrt{2}}{4} \\
p &= 7 \cos \left(\frac{\pi}{4}\right) = \frac{7\sqrt{2}}{2} \\
\phi &= \cos^{-1} \left(\frac{\sqrt{5}}{5}\right)
\end{align*}
\]

2. Find the indicated limits.

a.) \(\lim_\limits{t \to \pi} (\cosh(t - \pi), t, \sin t) = \langle 1, \pi, 0 \rangle\)

b.) \(\lim_\limits{t \to 3} \left(\frac{t^2 - 4}{t - 2}, e^{-\frac{1}{(t - 2)^2}}, \sin t\right) = \lim_\limits{t \to 3} \left(\frac{(t-2)(t+2)}{t-2}, e^0, \sin t\right) = \langle 6, 1, \sin 3 \rangle = \langle 9, 0, 3 \sin 2 \rangle\)
3. Find and sketch the curve of intersection for the following surfaces. 

\[ x^2 + y^2 = 4 \quad \text{and} \quad z = x^2 \]

\[ \begin{align*}
  x &= 2 \cos \theta \\
  y &= 2 \sin \theta \\
  z &= 4 \cos^2 \theta
\end{align*} \]
4. Find the length of the curve over the indicated interval.

\[ r(t) = (\sin t - t \cos t, \cos t + t \sin t, t^3), \quad t \in \left[0, \frac{\pi}{2}\right] \]

\[ \frac{d}{dt} \left( t^3 \right) = \left( \frac{\cos t - \cos t + \cos t \sin t, -\sin t + \sin t + \sin t \cos t, 2t} {t} \right) = \left( t \sin t, \cos t, \frac{2t}{t} \right) \]

\[ \| \frac{d}{dt} \| = \sqrt{t^2 \sin^2 t + \cos^2 t + \frac{4t^2}{t^2}} = \sqrt{t^2} \]

\[ L = \int_0^{\frac{\pi}{2}} \sqrt{t^2} \, dt = \frac{\sqrt{\pi}}{2} \cdot \frac{\pi^2}{6} = \frac{\sqrt{\pi^5}}{3} \]

5. Find the equation of the circle of curvature to the curve at the indicated point. Sketch both on the same axes.

\[ y = \ln x \text{ at } x = 1 \]

\[ y' = \frac{1}{x} ; \quad y'' = -\frac{1}{x^2} \]

\[ x' = \frac{1}{x^2 + 1} ; \quad x(0) = \frac{1}{2} \]

\[ y'(0) = 1 \Rightarrow \mathbf{N}(t) = \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) \]

\[ y(0) = 0 \Rightarrow \text{ pt. on curve: } (1, 0) \]

Center of circle: \( \frac{3}{2} \left( \frac{1}{2}, \frac{1}{2} \right) = \left( \frac{1}{2}, \frac{1}{2} \right) \)

Equation: \( (x - 3)^2 + (y + 2)^2 = \)