Exam 1
September 24th, 2008

Math 243 Section 450
Instructor: Patrick C. Rowe
Office: 323 Ewing Hall

Print Name: Answer Key

Academic Honesty Policy: Students at the University are expected to be honest and forthright in their academic endeavors. To falsify the results of one's research, to steal the words or ideas of another, to cheat on an examination, or to allow another to commit an act of academic dishonesty corrupts the essential process by which knowledge is advanced. It is the official policy of the University of Delaware that all acts or attempted acts of alleged academic dishonesty be reported to the Dean of Students Office.

By signing below, I acknowledge that I have read the above and that I have neither given nor received assistance on this examination.

Sign Name: Answer Key

All work must be shown to receive credit. Try to do all your work on this paper and clearly indicate your final answer. If you need additional paper, ask for it. All paper used must be turned in with your exam.

No Calculators may be used. All cellular phones and/or electronic devices of any kind must be turned off and put away. CHECK YOUR PHONE NOW!! If your phone rings during the exam, you may be asked to leave.

1
1. Find an equation of the sphere with center (6, 5, -2) and radius $\sqrt{7}$. Also describe the intersection the sphere makes with each of the three coordinate planes.

\[(x-6)^2 + (y-5)^2 + (z+2)^2 = 7\]

\[x=0 \] \text{xy-plane} \hspace{1cm} (y-5)^2 + (z+2)^2 = -29 \hspace{1cm} \text{no intersection}

\[y=0 \] \text{xz-plane} \hspace{1cm} (x-6)^2 + (z+2)^2 = -18 \hspace{1cm} \text{no intersection}

\[z=0 \] \text{xy-plane} \hspace{1cm} (x-6)^2 + (y-5)^2 = 3 \hspace{1cm} \text{circle with center (6, 5, 0) of radius}\sqrt{3}

2. If $v$ lies in the first quadrant ($v$ is 2d) and makes an angle of $\frac{\pi}{6}$ with the positive $x$-axis and $\|v\|=4$, find $v$ in component form.

\[\vec{v} = \langle 4 \cdot \cos \left( \frac{\pi}{6} \right), 4 \cdot \sin \left( \frac{\pi}{6} \right) \rangle = \langle 2, 2\sqrt{3} \rangle\]
3. Show that \( \|\mathbf{u} - \mathbf{v}\|^2 + \|\mathbf{u} + \mathbf{v}\|^2 = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2 \). This is called the parallelogram law.

\[
\|\mathbf{u} - \mathbf{v}\|^2 + \|\mathbf{u} + \mathbf{v}\|^2 = (\mathbf{u} - \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) + (\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} + \mathbf{v}) = \\
= \mathbf{u} \cdot \mathbf{u} - \mathbf{v} \cdot \mathbf{v} - \mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{u} + \mathbf{u} \cdot \mathbf{u} + \mathbf{v} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{v} + \mathbf{v} \cdot \mathbf{u} = \\
= 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2 - 2\mathbf{u} \cdot \mathbf{v} + 2\mathbf{v} \cdot \mathbf{u} = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2 = \\
= 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2 \quad \text{as required}
\]

4. Find the volume of the parallelepiped spanned by \((2, 1, 3), (3, -1, 2), (4, 0, 3)\).

\[
\begin{vmatrix}
2 & 3 & 4 \\
1 & 3 & 2 \\
-1 & 2 & 3
\end{vmatrix} = <5, 5, -5>
\]

\[
\begin{vmatrix}
2 & 0 & 3 \\
3 & 5 & -5
\end{vmatrix} = |20 + 0 - 15| = 5
\]

(This is the triple scalar product.)
5. Consider the following planes

\[2x - y + 3z + 4 = 0 \quad \text{and} \quad 3x + 2y + 5z + 2 = 0\]

and do the following.

a. Find parametric equations for the line of intersection.

b. Find an equation of a plane which is perpendicular to both planes and which passes through the origin.

c. The three planes you now have intersect at one point. What is it?

\[\begin{align*}
\text{a) } y &= 2x + 3z + 4 \quad \text{sub into other plane} \\
3x + 4x + 6z + 8 + 5z + 2 &= 0 \Rightarrow 7x + 11z + 10 = 0 \\
\Rightarrow x &= \frac{-11z - 10}{7} \\
2 &= t, \quad x = \frac{-11t - 10}{7}, \quad y = \frac{-22t - 20 + 3t + 4}{7} = \frac{-17t + 4}{7}, \quad z = t
\end{align*}\]

\[b) \text{ All plane } \perp \text{ to both planes must also be } \perp \text{ to their line of intersection i.e. its normal is the direction vector at the line. } \vec{n} = \left\langle -\frac{11}{7}, -\frac{1}{7}, 1 \right\rangle. \text{ To make the plane go through the origin, just set its constant term to zero. } \]

\[\begin{align*}
-\frac{11}{7}x - \frac{1}{7}y + z &= 0 \\
(\text{over})
\end{align*}\]
C. The line of intersection only intersects the plane in b. at one pt. We can find the pt by substituting the parameter eqns into the eqn for the plane.

\[-\frac{11}{7} (\frac{-11}{7} t - \frac{10}{7}) - \frac{1}{7} (\frac{-1}{7} t + \frac{8}{7}) + t = 0\]

\[\frac{121}{49} t + \frac{1}{49} t + t + \frac{110}{49} - \frac{8}{49} = 0\]

\[\frac{171}{49} t = \frac{-102}{49} \Rightarrow t = \frac{-102}{171}\]

Now substitute \(t\) back into the parameter eqns. (can leave like this)

\[\left(\frac{-11}{7}, \frac{-10}{7}, \frac{-1}{7}\right)\]

\[\left(\frac{121}{171}, \frac{110}{171}, \frac{136}{171}, \frac{-102}{171}\right)\]

\[\left(\frac{-58}{171}, \frac{1470}{171}, \frac{-102}{171}\right) = \left(\frac{-28}{57}, \frac{70}{57}, \frac{-34}{57}\right)\]
6. Sketch \( \frac{x^2}{4} + \frac{(y-1)^2}{4} + (z-1)^2 = 1 \).