Exam 2
July 19th, 2007
Math 243 Section 910
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Print Name: Answer Key

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By signing below, I acknowledge that I have read the above and that I have neither given nor received assistance on this examination.

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All work must be shown to receive credit. Try to do all your work on this paper and clearly indicate your final answer. If you need additional paper, ask for it. All paper used must be turned in with your exam.

No Calculators may be used. All cellular phones and/or electronic devices of any kind must be turned off and put away. CHECK YOUR PHONE NOW!! If your phone rings during the exam, you may be asked to leave.
Problem 1

a) Write an equation for the plane containing the points
   \[ P_1 = (0, 0, 4), P_2 = (1, 1, 3) \text{ and } P_3 = (3, 2, 0) \]

b) Find the distance between the plane from a) and the line given by
   \[ r(t) = (1, 2, 3) + t\langle 2, -1, -5 \rangle \]

c) Write an equation for a line which is perpendicular to the plane in a) and intersects the line in part b) at any point you choose.

d) Find the angle between the plane in part a) and the plane given by \( 8x + 3y - 2z + 2 = 0 \).

\[ \overrightarrow{v}_1 = \overrightarrow{P_1P_2} = \langle 1, 1, -1 \rangle ; \overrightarrow{v}_2 = \overrightarrow{P_1P_3} = \langle 3, 2, -4 \rangle \]

\[ \overrightarrow{v}_1 \times \overrightarrow{v}_2 = \begin{vmatrix} 1 & 3 & k \\ 1 & 1 & -1 \\ 3 & 2 & -4 \end{vmatrix} = \langle -2, 1, -1 \rangle = \overrightarrow{n} \]

\[ -2x + y - (z - 4) = 0 \]

b) Plane is \( \perp \) to plane

\[ \overrightarrow{v}_1 \times \langle 2, -1, -5 \rangle = \begin{vmatrix} 1 & 3 & k \\ 1 & 1 & -1 \\ 2 & -1 & -5 \end{vmatrix} = \langle -2, 3, -5 \rangle \]

\[ \overrightarrow{w} = \langle 1, 2, -1 \rangle \text{ (vector from } P_1 \text{ to } (1, 2, 3) \rangle \]

\[ |\overrightarrow{n}| = \sqrt{4 + 1 + 1} = \sqrt{6} \]

\[ d = \frac{|\overrightarrow{w} \cdot \overrightarrow{n}|}{|\overrightarrow{n}|} = \frac{-2 + 2 + 5}{\sqrt{6}} = \frac{5}{\sqrt{6}} \]
c) Choose \((1, 2, 3)\)
\[ \vec{z}(t) = \langle 1, 2, 3 \rangle + t \langle -2, 1, -1 \rangle \]

d) \[ 8x + 3y - 2z + 2 = 0 \]
\[ \vec{n} = \langle -2, 1, -1 \rangle \]
\[ \vec{n}_2 = \langle 8, 3, -2 \rangle \]
\[ |\vec{n}_2| = \sqrt{64 + 9 + 4} = \sqrt{77} \]
\[ |\vec{n}| = \sqrt{6} \]

\[ \Theta = \cos^{-1} \left( \frac{-16 + 3 + 2}{\sqrt{77} \sqrt{6}} \right) = \cos^{-1} \left( \frac{-1}{\sqrt{462}} \right) \]
Problem 2

For each of the following vector-valued functions, state the domain.

a) \( \mathbf{r}(t) = \langle \ln(t), \sqrt{t-1}, t^3 \rangle \quad t \in [1, \infty) \)

b) \( \mathbf{r}(t) = \langle t, \tan(t-1), \sin(t) \rangle \quad t \in \mathbb{R} \setminus \left\{ \frac{2n\pi}{1 \pm 1} \right\} + 1 \)

Problem 3

For the following vector-valued functions, find the indicated limits.

a) \( \lim_{t \to 1} \left( \sqrt{t} + \ln(t) \frac{t}{t-1} \mathbf{j} + 2t^3 \mathbf{k} \right) = \left( \frac{1}{e} \mathbf{i} + \frac{1}{e^2} \mathbf{j} + \frac{1}{e^3} \mathbf{k} \right) \)

b) \( \lim_{t \to -\infty} \left( e^{-t} \mathbf{i} + \frac{1}{t} \mathbf{j} + \frac{t}{t+1} \mathbf{k} \right) = 0 \)
Problem 4

Sketch the curve given by

\[ \mathbf{r}(t) = (\cos(t), \cos(t), \sin(t)) \]

and indicate the direction the curve is traced out as \( t \) increases.

- the curve is an ellipse drawn on the plane \( y = x \).
Problem 5

Find the curve of intersection of the following surfaces:

\[ x^2 + y^2 = 4 \]
\[ z = x^2 \]

From the 1st surface:
\[ x = 2 \cos \theta \]
\[ y = 2 \sin \theta \]

Put into the 2nd surface:
\[ z = 4 \cos^2 \theta \]
\[ x^2 + y^2 = 4 \]

\[ (2 \cos \theta, 2 \sin \theta, 4 \cos^2 \theta) \]