Exam 3
January 30th, 2009

Math 242 Section 013
Instructor: Patrick C. Rowe
Office: 323 Ewing Hall

Print Name: Answer Key

Academic Honesty Policy: Students at the University are expected to be honest and forthright in their academic endeavors. To falsify the results of one's research, to steal the words or ideas of another, to cheat on an examination, or to allow another to commit an act of academic dishonesty corrupts the essential process by which knowledge is advanced. It is the official policy of the University of Delaware that all acts or attempted acts of alleged academic dishonesty be reported to the Dean of Students Office.

By signing below, I acknowledge that I have read the above and that I have neither given nor received assistance on this examination.

Sign Name: Answer Key

All work must be shown to receive credit. Try to do all your work on this paper and clearly indicate your final answer. If you need additional paper, ask for it. All paper used must be turned in with your exam.

No Calculators may be used. All cellular phones and/or electronic devices of any kind must be turned off and put away. CHECK YOUR PHONE NOW!! If your phone rings during the exam, you may be asked to leave.
1. If the geometric series converges, find the sum. Otherwise state that it diverges.

a. \( \sum_{n=0}^{\infty} \left( \frac{5}{4} \right)^n \)  
   \[ r = \frac{5}{4} > 1 \]  
   Series diverges

b. \( \sum_{n=0}^{\infty} \left( \frac{3}{4} \right)^n \)  
   \[ r = \frac{3}{4} < 1 \]  
   Converges with sum \( S = \frac{1}{1 - \frac{3}{4}} = \frac{1}{\frac{1}{4}} = 4 \)

c. \( \sum_{n=0}^{\infty} \left( \frac{2}{3} \right)^n + \left( \frac{2}{3} \right)^n \)  
   \[ r = \frac{2}{3} > 1 \]  
   Series diverges

\[ \sum_{n=0}^{\infty} \left( \frac{2}{3} \right)^n + \left( \frac{2}{3} \right)^n = \sum_{n=0}^{\infty} \left( \frac{2}{3} \right)^n + \sum_{n=0}^{\infty} \left( \frac{2}{3} \right)^n \]

\[ = \sum_{n=0}^{\infty} \left( \frac{2}{3} \right)^n + 4 = 2 + 4 = 6 \]

\[ r = \frac{2}{3} \Rightarrow S = \frac{1}{1 - \frac{2}{3}} = 2 \]
2. Use the integral test to determine if the series converges or diverges.

a. \[ \sum_{n=1}^{\infty} n e^{-n} \]
   \[ f(x) = xe^{-x} \]
   \[ \int_{1}^{\infty} xe^{-x} \, dx = \left. -xe^{-x} - e^{-x} \right|_{1}^{\infty} = \frac{2}{e} \]
   Use the integral test.
   \[ \therefore \text{Series converges} \]

b. \[ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \]
   \[ f(x) = \frac{1}{\sqrt{x}} \]
   \[ \int_{1}^{\infty} \frac{1}{\sqrt{x}} \, dx = \left. \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right|_{1}^{\infty} = \infty \]
   \[ \therefore \text{Series diverges} \]

3. For the following series, use either the direct comparison, limit comparison, or alternating series test to determine convergence or divergence.

a. \[ \sum_{n=1}^{\infty} \frac{n}{2n^3 + 1} \]
   \[ \text{Direct comparison to } \frac{1}{n^2} \]
   \[ \frac{n}{2n^3 + 1} \leq \frac{1}{2n^2} = \frac{\frac{1}{2}}{n^2} \leq \frac{1}{n^2} \]
   \[ \therefore \text{Series converges} \]

b. \[ \sum_{n=0}^{\infty} \frac{1 + \sin(n)}{10^n} \]
   \[ \text{Direct comparison to } \frac{2}{10^n} \]
   \[ \frac{1 + \sin(n)}{10^n} \leq \frac{2}{10^n} \]
   \[ \therefore \text{Series converges} \]
c. \( \sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n} \)

Use Alternating Series Test

\[
\lim_{n \to \infty} \frac{n}{n \ln n} = \lim_{n \to \infty} \frac{1}{\ln n} = 0
\]

\( \therefore \) Series converges

\[
\lim_{n \to \infty} \frac{1}{n} = 0
\]

\( \therefore \) Series diverges

d. \( \sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n} \)

\[
\lim_{n \to \infty} \frac{\ln n}{n} = \lim_{n \to \infty} \frac{1}{n} = 0
\]

\( \therefore \) Series converges

4. For the following series determine if the series diverges, converges absolutely, or converges conditionally.

a. \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}} \)

The absolute series is \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \). Apply the integral test:

\[
\int_{1}^{\infty} \frac{dx}{\sqrt{x}} = \left[ 2 \sqrt{x} \right]_{1}^{\infty} = \infty
\]

\( \therefore \) Divergent. But the alternating series test:

\[
\lim_{n \to \infty} \frac{1}{\sqrt{n}} = 0
\]

\( \therefore \) Conditionally convergent

b. \( \sum_{n=1}^{\infty} \frac{\sin(4n)}{4^n} \)

The absolute series is \( \sum_{n=1}^{\infty} \frac{1}{4^n} \).

By direct comparison:

\[
\frac{\left| \sin(4n) \right|}{4^n} \leq \frac{1}{4^n}
\]

\( \therefore \) convergent. \( \therefore \) absolutely convergent
c. \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \); The absolute series is \( \sum_{n=1}^{\infty} \frac{\tan(n\pi/2)}{n^2} \).

By direct comparison, \( \frac{\tan(n\pi/2)}{n^2} \leq \frac{\pi^2}{n^2} \): convergent \Rightarrow \text{absolutely convergent.}

5. Find the radius and the interval of convergence.

a. \( \sum_{n=1}^{\infty} \frac{\ln(n)}{n^3} \): \( \lim_{n \to \infty} \frac{\ln(n+1)}{\ln n} \) = \( \frac{\ln 1}{\ln 1} \) = 1. Check the endpoints \( X = 1 \): \( \sum_{n=1}^{\infty} \frac{\ln(n)}{n^3} \) divergent and \( X = -1 \): \( \sum_{n=0}^{\infty} \frac{\ln(n)}{n^3} \) divergent. \( R = 1, I = (-1, 1) \)

b. \( \sum_{n=1}^{\infty} \frac{(3x - 2)^n}{n^3} \): \( \lim_{n \to \infty} \left| \frac{(3x - 2)^{n+1}}{(3x - 2)^n (n+1)^3} \right| = \left| \frac{3x - 2}{n} \right| \Rightarrow \frac{3}{3} |x - \frac{2}{3}| < 1 \Rightarrow \frac{2}{3} < x < \frac{5}{3} \). Check the endpoints: \( |x - \frac{2}{3}| = 1 \Rightarrow x = \frac{1}{3}, 1 \Rightarrow \text{convergent} \) and \( |x - \frac{5}{3}| = 1 \Rightarrow x = -1, \frac{7}{3} \Rightarrow \text{convergent} \) alternating harmonic series \( \Rightarrow \text{convergent} \). \( R = 1, I = [\frac{1}{3}, \frac{7}{3}] \)

6. Write the repeating decimal as an infinite series.

\( 0.73 \frac{\overline{73}}{100} \)

\( \frac{73}{100} + \frac{73}{100^2} + \frac{73}{100^3} + \cdots = \sum_{n=1}^{\infty} \frac{73}{100^n} \) = \( \frac{73}{100} \sum_{n=1}^{\infty} \frac{1}{100^n} = \frac{73}{100} \frac{1}{1 - \frac{1}{100}} = \frac{73}{100} \cdot \frac{1}{99} = \frac{73}{99} \)