Exam 1
January 13th, 2009
Math 242 Section 013
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Print Name: Answer Key

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By signing below, I acknowledge that I have read the above and that I have neither given nor received assistance on this examination.

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All work must be shown to receive credit. Try to do all your work on this paper and clearly indicate your final answer. If you need additional paper, ask for it. All paper used must be turned in with your exam.

No Calculators may be used. All cellular phones and/or electronic devices of any kind must be turned off and put away. CHECK YOUR PHONE NOW!! If your phone rings during the exam, you may be asked to leave.
1. Find the following limits by any method you wish.

a. \( \lim_{x \to 1} \frac{x^2 - 1}{x^2 - 1} = \lim_{x \to 1} \frac{ax^2 - a}{bx^2 - b} = \frac{9}{6} \)

b. \( \lim_{x \to 0} (1 - \tan x) \sec x = 0 \cdot \sqrt{2} = 0 \)

c. \( \lim_{z \to \infty} \frac{e^z}{z^3} = \lim_{x \to \infty} \frac{e^x}{5x^2} = \lim_{x \to \infty} \frac{e^x}{6x} \neq \lim_{x \to \infty} \frac{e^x}{6} = \infty \)
\[ \lim_{x \to 4^+} (3x - 12)^{x-4} = y \Rightarrow \ln y = \lim_{x \to 4^+} (x-4) \ln (3(x-4)) = \lim_{x \to 4^+} \frac{\ln(3(x-4))}{x-4} \cdot \frac{3}{3} = \lim_{x \to 4^+} \frac{3(x-4)}{x-4} = 3 \Rightarrow \ln y = 0 \Rightarrow y = 1 \]

\[ \lim_{x \to 2^+} \left( \frac{8}{x^2 - 4} - \frac{x}{x - 2} \right) = \lim_{x \to 2^+} \frac{8 - x(x+2)}{x^2 - 4} = \frac{8 - 6}{2} = \frac{2}{2} = 1 \]

\[ \lim_{x \to \infty} x \sin \left( \frac{\pi}{x} \right) = \lim_{x \to \infty} \frac{\sin \left( \frac{\pi}{x} \right)}{\frac{1}{x}} = \lim_{x \to \infty} -\frac{\pi \cos \left( \frac{\pi}{x} \right)}{x^2} = \frac{\pi}{\infty} = 0 \]
3. Find the area between $y = \sin x$ and $y = \frac{2}{\pi} x$ on the interval $x = [0, \pi]$.

\[\sin x = \frac{2}{\pi} x \implies x = \frac{\pi}{2}\]

\[
\int_0^{\pi/2} \sin x - \frac{2}{\pi} x \, dx + \int_{\pi/2}^\pi \frac{2}{\pi} x - \sin x \, dx =
\]

\[= -\cos x - \frac{x^2}{\pi} \bigg|_0^{\pi/2} + \frac{x^2}{\pi} + \cos x \bigg|_\pi^{\pi/2} =
\]

\[= \left( -\frac{\pi}{4} + 1 \right) + \left( \pi - 1 - \frac{\pi}{4} \right) = \frac{\pi}{2}\]
4. Verify that the shell method and the washer method calculate the same volume for the following problem.

Rotate the region bounded by \( y = x \) and \( y = \sqrt{x} \) about the y-axis

**Washer method**

\[
\pi \int_{0}^{1} y^2 - y^4 \, dy = \pi \left( \frac{y^3}{3} - \frac{y^5}{5} \right) \bigg|_{0}^{1} = \pi \left( \frac{1}{3} - \frac{1}{5} \right) = \\
\pi \cdot \frac{2}{15} = \frac{2\pi}{15}
\]

**Shell method**

\[
2\pi \int_{0}^{1} x(\sqrt{x} - x) \, dx = 2\pi \int_{0}^{1} x^{3/2} - x^2 \, dx = 2\pi \left( \frac{2}{5}x^{5/2} - \frac{x^3}{3} \right) \bigg|_{0}^{1} = \\
2\pi \left( \frac{2}{5} - \frac{1}{3} \right) = 2\pi \left( \frac{1}{15} \right) = \frac{2\pi}{15}
\]
5. Find the average value of \( f(x) = \sin 4x \) on \([-\pi, \pi]\)

\[
\bar{f}(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \sin 4x \, dx = \frac{1}{2\pi} \left[ -\frac{1}{4} \cos 4x \right]_{-\pi}^{\pi} = \frac{1}{8\pi} (1 - 1) = 0
\]

\[
\bar{f}(x) = 0
\]

6. Calculate the indefinite integrals by any method you can.

a. \( \int \frac{2x}{e^x} \, dx = 2 \int x e^{-x} \, dx = 2 \left( -xe^{-x} + \int e^{-x} \, dx \right) = -2xe^{-x} - 2e^{-x} + C \)

b. \( \int \sin^2 x \cos^2 x \, dx = \frac{1}{4} \int (1 - \cos 2x)(1 + \cos 2x) \, dx = \frac{1}{4} \int 1 - \cos^2 2x \, dx = \frac{1}{4} \int 1 - \frac{1}{2} \cos 4x \, dx = \frac{1}{8} \int 1 - \cos 4x \, dx = \frac{1}{8} x - \frac{1}{32} \sin 4x + C \)
c. \[ \int \frac{1}{\sqrt{1-4x^2}} \, dx \]
\[ x = \frac{1}{2} \sin \theta \Rightarrow dx = \frac{1}{2} \cos \theta \, d\theta \]
\[ \frac{1}{2} \int \frac{1}{\sqrt{1-\sin^2 \theta}} \cos \theta \, d\theta = \frac{1}{2} \int \cos^2 \theta \, d\theta = \frac{1}{2} \int 1 + \cos 2\theta \, d\theta = \]
\[ = \frac{1}{4} \theta + \frac{1}{8} \sin 2\theta + C = \frac{1}{4} \theta + \frac{1}{4} \sin \theta \cos \theta + C = \]
\[ = \frac{1}{4} \ln \left( \frac{2x}{\sqrt{4x^2-1}} \right) + \frac{1}{4} \cdot 2x \cdot \sqrt{4x^2-1} + C = \]
\[ = \frac{1}{4} \ln \left( \frac{2x}{\sqrt{4x^2-1}} \right) + \frac{1}{2} \cdot x \cdot \sqrt{4x^2-1} + C \]

\[ \int \frac{x-9}{(x+5)(x-2)} \, dx \]

Use partial fractions:
\[ \frac{x-9}{(x+5)(x-2)} = \frac{A}{x+5} + \frac{B}{x-2} \Rightarrow x-9 = A(x-2) + B(x+5) \]
\[ x=2: B = -1 \]
\[ x=-5: A = 2 \]

Integral becomes:
\[ \int \frac{2}{x+5} + \frac{-1}{x-2} \, dx = 2 \ln |x+5| - \ln |x-2| + C = \]
\[ = \ln \left| \frac{x+5}{x-2} \right| + C \]