Maple Project 1

In this session we will have fun with functions and their inverses. Suppose we wanted to determine whether \( y = 6 - 2x^2 \) is 1-1 or not. One way to do it is just use the horizontal line test. We can use Maple to plot the graph over some interval and then we can apply the test.

\[
\text{> plot}(6-2*x^2,x=-5..5);
\]

So we see that \( y = 6 - 2x^2 \) is not one-to-one since it clearly fails the horizontal line test on the interval \([-5,5]\]. Huzzah! Let's plot it on a smaller interval, say \([1,3]\):

\[
\text{> plot}(6-2*x^2,x=1..3);
\]

Our friend clearly passes the horizontal line test on the interval \([1,3]\). Let's find its inverse on this interval. First, let's define it as a function:

\[
\text{> f:=x->(4*x-5)/(x+3);}
\]

Remember, to find the inverse of \( y = f(x) \), we need to solve for \( x \) as a function of \( y \). In the following command, we solve for \( x \) as a function of \( y \) and put the answer in the variable called "solution":

\[
\text{> solution:=solve(y=f(x),x);}
\]

So we see either \( x \) is positive or negative. In our case, we want \( x \) to be positive since we are in the interval \([1,3]\), so we would like the first solution. We can refer to it with:

\[
\text{> solution[1]};
\]

(When finding inverses, you don't always get two solutions. In your project, you will only get one solution, so you won't have to worry about the command we just did.) So, continuing like we did in class, we can rewrite it by substituting \( x \) for \( y \). We do this by using the "subs" command.

\[
\text{> subs(y=x,solution[1]);}
\]

Let's take this last output and make it into a function we can actually plug values into. We do this by using the "unapply" command and our friend ",", which always refers us to our last output:

\[
\text{> g:=unapply(%,x);} 
\]

So, this function "g" is indeed the inverse of \( f \). Now we can plot \( y = f(x) \), its inverse \( y = g(x) \), and the line \( y = x \) all on the same graph by using the plot command. The braces just mean that all 3 curves will be on the same plot. I also added in the "y=0..6" to restrict the y values between 0 and 6, so we could see the plots a little better. We are also plotting these graphs on the interval \([0,6]\).

\[
\text{> plot}([f(x),g(x),x],x=-2..3);
\]

Doesn't look bad, eh? The green curve is \( y=f(x) \), the red one is \( y=x \), and the yellow one is \( y=g(x) \). Remember that the graphs of \( f \) and \( f \) inverse should be symmetrical about the line \( y=x \), i.e. one should be the reflection of the other across the line \( y=x \).

Now let's find \( g'(2) \) using the inverse differentiation theorem. The formula is: \( g'(2) = 1 / f'(g(2)) \), where \( g \) is \( f \) inverse. We really only need to find the derivative of \( f \) and then plug things in. This command finds the derivative of \( f \):

\[
\text{> diff(f(x),x);} 
\]

Here, our derivative is just an expression. Let's make it into a
function we can plop values into by using that "unapply" thingy again.
> fprime:=unapply(%,x);
Now lets plug things into our formula; so we see that g'(2) is equal
to:
> 1/fprime(g(5));
Supposedly this is g'(2), the derivative of f inverse at 2. Let's
doublecheck by actually taking the derivative of g (which is f inverse)
and evaluating it at 2, instead of using the formula.
> diff(g(x),x);
> gprime:=unapply(%,x);
> gprime(5);
> simplify(g(f(x)));
Well, there you have it. The formula works, darn it all!

So here's the project problems. Do them using Maple and the techniques
we have valiantly pioneered in the above section. (Start a new
worksheet by choosing "New" from the "File" menu. When you are
finished, print out your Maple sheet and PUT YOUR NAME ON IT, and give
it to me.) You can put text into your worksheet by clicking on the "T"
button in the menu up top. When you are done, click on the button to
the right of it to get the Maple prompt back for your Maple commands.

1) Graph y = x^2 + x. Is it one-to-one? Why or why not? If not,
find an interval on which the function is one-to-one.
2) a) Find the inverse of f(x) = (4x-5)/(x+3).
   b) Plot f, its inverse, and the line y=x all on the same graph over
      the interval [-2,3].
   c) Verify the inverse differentiation theorem for this function at
      x=5.