Contemporary Applications of Mathematics (Math512 sec. 010 & 080) **Fall 2004**
MWF 1430-1520 Mem 109
WWW: `http://www.math.udel.edu/~rossi/Math512`

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Background

Mathematical modeling is the never-ending activity of determining, deriving, deducing or just plain guessing a mathematical structure that describes something. You can try to model the motion of a spinning disk of pizza dough, the stock market, or the spread of rumors in a small town. Modeling requires that one knows some mathematics and something about the problem. While most mathematics courses involve techniques for solving equations or examining the properties of mathematical structures, this course focuses on the determination, refinement and understanding of mathematical structures that describe a problem of interest.

Your objective

You and your teammates will explore an open problem mathematically and produce a report describing your activities modeling the problem. Your final report must form connections between a concrete problem with an underlying mathematical structure. These connections may take many forms including

- Exact solutions to a system of equations that describes the problem.
- Convergent or asymptotic approximations to solutions of equations that describe the problem.
- Reduced mathematical properties such as a theorem about mathematical structures that describe the problem.
- Careful numerical investigations and simulations of equations that describe the problem.
- Generalizations to other seemingly unrelated problems or categories of problems that share the same mathematical structure.

This course is problem-driven and data-driven which means that students are expected to get involved in their problem by reading about it, learning about it, observing it and collecting data.

Your resources

All of the following will help you achieve your objectives:

- Time: Attendance in this course is mandatory.
- The MEC lab in in Ewg101C. The **M**odeling, **E**xperimentation and **C**omputation **lab**oratory is a unique facility for students to conduct small, careful experiments. In short, this is a place where you can observe a process and gather your own data.
- Office hours: Mon 1300-1400, Tue 1000-1100 and Fri 1115-1215.

  Office hours are one of the most valuable and least used resources at the University, and I hope you will take advantage of them. I want to help you learn this material, so do not be shy about seeing us outside of class. If you need to see me at a time other than an office hour, feel free to "drop in" or make an appointment.

- Your classmates: Math is not a competitive sport. There are many obvious reasons to work together. Even if you end up helping others most of the time, teaching is one of the best ways to gain a deeper understanding of a subject.
- Textbooks & library research: There is no required textbook for this class, but you will want to keep a few books handy. First, if your fundamentals are a bit rusty, you should dust off your calculus textbook. Next, a standard text on ordinary differential equations and linear algebra will definitely be useful. Also, I expect students to visit the library frequently.

Course structure

The structure of Math512 is designed to support student project teams in their investigations. All students will be required to participate in all classroom activities. This means that every student must be involved in all projects to a limited extent, and every student must be an expert on all aspects of their own particular project.

The course will have frequent **mini-lectures** on mathematical topics that are relevant to the projects. Modest **problem sets** will be assigned to reinforce this material. To make the most of your time and effort, you may substitute problems in the problem sets for problems that are relevant to your projects.

Beginning in the fourth week, Mondays will be reserved for **presentations** by the teams. One speaker from each team will update the class on the problem and its current status. The presenter will be randomly selected. Students will be expected to ask questions or make comments on what is presented. You will be graded on your presentations and on your participation in discussions during presentations by other teams.

By midnight on the Sunday prior to each presentation, each team will post a **weekly planning report** which has two sections, and the presentation should direct address the weekly planning report. The report is an itemized list, and planning items should be specific activities. Examples would include:

- “Next week, we will perform three experiments with low, medium and high concentrations of X to see if it has any impact on measurements of Y.”
- “Next week, we are going to debug our code that simulates X and test it with problem Y.”

Non-examples would include:

- “Next week, we are going to the MEC lab and try the experiment.”
- “Next week, we are going to go to the library to learn about our problem.”
- “Next week, we are going to use Maple to solve our problem.”

Examples of items that can and should go into a planning report are:

- Plans to acquire information from a passive source such as the library and the web.
- Plans to acquire information from a human source such as faculty, other students or an outside expert.
- Plans to make experimental observations and measurements.
- Development of assumptions that either must be made or are convenient to make to advance the project.
- Plans to develop or refine a mathematical model.
- Plans to analyze or manipulate a mathematical model.
- Plans to develop computational tools to study the problem.
- Plans to write or revise portions of the final report.

Every student will maintain a **laboratory notebook** in this course. This notebook is part of your project in the sense that everything that goes into your final project report should come from your notebook. Periodically, we will collect notebooks and examine them. The notebooks should document all activities related to your projects. It is OK to be wrong. It is OK to hit dead ends. It is NOT OK to forget what you have already tried or have to reproduce work because you did not record it properly. In addition to whatever you write, your notebook may include summaries of journal articles, plots or other computer output, photographs and other items that will provide a record of your investigation.

Project milestones are designed to help each team make organized and coherent progress in a major undertaking. Each milestone marks a different stage in your progression toward a final project that will be the culmination of the course. Each milestone is a written report outlining the problem and the team's progress and prospects. Grades will be assigned differently for different milestones. For instance, at the first milestone, we will pay careful attention to your literature review and we will also look over your lab notes. However, at this early time, we would not expect that anyone will have made significant progress analyzing their model yet. In fact, some might not even have a model at this point.

Milestone	Lit. review Assumptions Definitions Formulation	Analysis Model solutions Measurements Parameter estimation	Simulations Comparisons Strengths & Weaknesses Synthesis	Laboratory notes Style Brevity Clarity Presentation
1	80%	5%	0%	15%
2	20%	60%	0%	20%
3	5%	40%	25%	30%
4	0%	40%	20%	40%
5	0%	20%	30%	50%

Grading

Your final grade is a weighted average of the constituent components in this course.

Milestones 1-5	75% (15% each)
Problem sets	10%
In-class presentations	7%
Weekly planning report	4%
Participation	4%

Final letter grades will be assigned based on the following percentages of your total point score:

$$100 \leftarrow \text{A} \rightarrow 93 \leftarrow \text{A-} \rightarrow 90 \leftarrow \text{B+} \rightarrow 87 \leftarrow \text{B} \rightarrow 83 \leftarrow \text{B-} \rightarrow 80 \leftarrow \text{C+} \rightarrow 77 \leftarrow \text{C}$$
$$\text{C} \rightarrow 73 \leftarrow \text{C-} \rightarrow 70 \leftarrow \text{D+} \rightarrow 67 \leftarrow \text{D} \rightarrow 63 \leftarrow \text{D-} \rightarrow 60 \leftarrow \text{F} \rightarrow 0$$

We reserve the right to adjust this scale to improve grades if the course material proves to be unreasonably demanding.

Tentative topics for mini-lectures

All of these topics will be motivated by the problems in the course and examples from fluid mechanics, electricity and magnetism, heat transfer, reaction-diffusion systems and combinations of all of these.

- Conservation laws. Variational methods, mechanics, Euler-Lagrange equations. Review of Gauss' and Stokes' Theorems and relevance to conservation laws.
- Scaling, dimensional analysis and dimensionless constants: Converting units, scale experiments, dynamic similarity...
- Working with data I. Interpolation, least squares, parameter estimation, splines, error analysis and reduction.
- Conservations laws: Partial differential equations, flux, diffusion, wave propagation, Helmholtz equation, traffic, pipe flows...
- Working with data II. Assessing the quality of a model. Norms and metrics.
- Numerical methods for ODEs.
- Numerical methods for PDEs.

Tentative schedule

Week	Event
1 Sep	Introduction, project presentations.
6 Sep	MEC Lab tour, lab safety, student project bidding.
13 Sep	Tools of the trade: Research, writing and computation. Milestone #1.
20 Sep	Conservation laws, non-dimensionalization/scaling, similitude, Buckingham's Π Theorem.
27 Sep	Working with data: mean, variance, parameter estimators, best fits, interpolations.
4 Oct	Fourier methods for PDEs: the trackshoe guide, Milestone #2.
11 Oct	Fourier methods for PDEs: the trackshoe guide (cont'd)
18 Oct	Linear stability analysis and calculations.
25 Oct	Writing, poster and verbal presentation workshop. Milestone #3.
1 Nov	Useful software tools and tricks, numerical methods.
8 Nov	Numerical methods (cont'd).
15 Nov	Variational methods. Milestone #4.
22 Nov[1]	Variational methods (cont'd).
29 Nov	Miscellany.
6 Dec	Milestone #5.

Important dates

14 Sep	No fee drop/add deadline
26 Oct	W withdrawal deadline
28 Nov	No class
8 Dec	Last day of classes

[1]There will be no class on 26 Nov due to the Thanksgiving Holiday