

# Swarm Behavior

Alpha Swarm

Gloria Amakobe, Karol Grabczewski, Tapan Patel

December 6, 2006

## 1 Introduction

It is quite hard to find animals in nature that do not socialize (aggregate) for one reason or another. A large group offers protection from a predator since larger group size decreases the chances of being caught. Furthermore, as a group, individuals can perform evasive maneuvers to outwit a predator. For example, upon predator threat, groups can explode into individuals or form a tight cluster, both designed to confuse the predator. Of course, animals aggregate even when there is no predator threat since highly coherent, aligned arrangement offers aerodynamic advantages.

Whatever the reason might be for group formation, the details of this behavior are important. Yet despite the near universality of aggregation, mathematicians and biologists have only recently begun to probe its underlying mechanisms. The implications for aggregate behavior extend far beyond an understanding of animal behavior. The same laws governing animal behavior can be generalized to guide artificial machines or robots to carry out complex routines by relying only on local interactions. One could argue that this is perhaps the ultimate goal of studying animal aggregations so that one day, we can create robot aggregates that can mimic the predator detection, predator evasion and target seeking activities of animal groups.

There has been a strong effort both on computational and mathematical front to model animal group behavior. Reynolds offers a way to computationally model the motion of an individual primarily by a seek and flee behavior, and more complex behaviors are modeled by combining these two primitive behaviors (seek/flee) in some fashion [3]. In a more mathematical approach, Couzin improves on Reynold's work by simulating the behavior of individuals as the result of local repulsion, alignment and attractive tendencies (formally introduced as three zones of attraction, repulsion and orientation) [2]. This approach is logical since for many animals, the presence of, and distance to other animals is important in determining their spatial distribution by social interactions. By varying the relative size of these zones, the group can take on different behaviors, ranging from milling to torus to translation, etc.

One of the more practical applications of swarming is using robots to map a boundary of an unknown and dangerous region. We investigate whether the boundary detection can be more efficient using the framework of local interactions present in a swarm system. This problem can be solved by a group of single individuals; however, collective mentality should provide a more efficient and robust solution.

Another interesting type of animal behavior is the role of leadership in a group. In a group of several hundred animals, for example honeybees, only a few percentage of the population is informed about the location of new food source. Since the knowledgeable members (leaders) are indistinguishable from the rest of the group (followers), the question remains how can only a few leaders efficiently lead an entire group to some final destination? That is, how does information travel over long distances through only local and social interactions. Furthermore, what if there is a predator along the way? The leaders now have the added difficulty of not only leading a cohesive group but that group could fragment at any point due to predator encounter. We investigate these two types of behavior in the present study. The main issues at hand for the predator model were: what is the best strategy to avoid predator - will (or should) the group fragment when the predator is detected? Once predator threat is gone, will the group reform and head toward the target? Will the reaction to predator propagate throughout the group naturally, without having to code in any extra behavior? and, What role will leader size have in this framework?

Ultimately, we are investigating if a local mathematical framework can control group behavior to perform a variety of unique tasks with minimal human input.

## 2 Assumptions

We impose several assumptions and restrictions on each member of the entire swarm in order for the tasks to be properly accomplished; however, we try to minimize direct control of the swarm as a unit. As done in Couzin's article, we rely on local interactions as a function of the distance between members to determine the immediate behavior of an individual. Because of this, we assume every animal has a zone of repulsion, orientation, and attraction, as mentioned in the literature [2]. The zone of repulsion defines an area where individuals are not comfortable with other individuals to enter and therefore will move in order for this minimal safe distance to be preserved at all times. The zone of orientation is an area in which the individuals will orient themselves to the direction of their closest neighbor. The zone of attraction is a region in which individuals are pulled to their neighbors. Lastly, in the case of predator, we assume another zone, the zone of predator avoidance (zop) which is defined such that if a predator is within this zone, the swarm's first priority is to flee. The exact description of what it means to flee is discussed later. In our representation, each individual is considered a point mass, occupying no volume, and flying through an idealized vacuum, where it do not experience any outside forces, such as gravity, drag, friction, electrostatics, or any other force outside our definition. The only force an individual experiences is the direct result from social interactions and environmental stimulus.

In the presence of a boundary, we assume that every individual of the swarm knows the

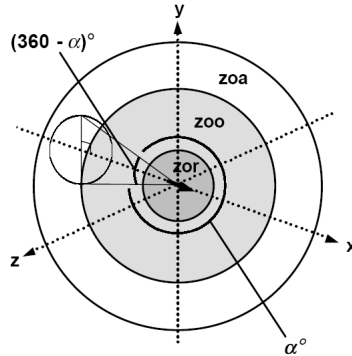


Figure 1: Representation of the three zones of interaction in the model for an individual centred at the origin.  $zor$  = zone of repulsion,  $zoo$  = zone of orientation,  $zoa$  = zone of attraction. The blind spot behind an individual is shown with  $\alpha$  = field of perception. Figure adopted from [2]

value of the function defining the boundary,  $f(x, y, z)$ , and can read the gradient of boundary function.

We also assume that only the leaders have information about an outside goal or destination, such that the knowledge of that goal can only be transmitted to the group by the leaders' movements. Furthermore, the predator is an all powerful being, moving faster than any individual in our group and capable of killing multiple individuals simultaneously. The predator does not stop until the entire swarm has been killed.

### 3 The Mathematical Model

We decided to work in an artificial physics (AP) framework in which physical forces drive a swarm system to a final configuration or state. The desired configuration is one that minimizes overall system potential energy while performing some task (such as getting to a target while avoiding a predator). Such a framework is especially appropriate since AP provides distributed control of large collections of agents that react to artificial forces (motivated by natural physical forces). This framework has been useful in the past for providing an effective mechanism for studying self-assembly, fault-tolerance and self-repair [4].

#### 3.1 Forces Governing Local Interactions

Each individual  $i$ , is represented by the position vector  $\mathbf{x}_i$ , velocity vector  $\dot{\mathbf{x}}_i$ , and an acceleration vector  $\ddot{\mathbf{x}}_i$ . Using a discrete time approximation to the continuous behavior of the group with a time step  $\delta t$ , at each time step, a new acceleration is calculated. Since we assumed unit mass for each individual, the acceleration is simply the sum of all the forces on

that individual (eq 1). New velocity and position vectors can then be determined by simple kinematic relations.

$$m\ddot{\mathbf{X}} = \mathbf{F}_i + \mathbf{F}_a + \mathbf{F}_e, \quad (1)$$

where  $F_i$  is an orientation force,  $F_a$  is an interactive force (with both attractive and repulsive terms), and  $F_e$  is an external stimulus force. The functional forms of these forces are:

1. The interactive force is defined as :

$$\mathbf{F}_a = \sum_{j=1}^N f(\phi_{i,j}) 4\epsilon \left( 12 \frac{\sigma^{12}}{r^{13}} - \frac{\sigma^6}{r^7} \right) \hat{\mathbf{r}}, \quad (2)$$

where  $r$  is the distance between two individuals,  $\hat{\mathbf{r}}$  is a unit vector in the direction of the neighbor,  $\epsilon$  is proportional to strength of attraction and  $\sigma$  is the strength of repulsion between individuals.

The function  $f(\phi_{i,j})$  is defined as :

$$f(\phi_{i,j}) = 1 - \left( \frac{1 - \cos(\phi_{i,j})}{2} \right)^2, \quad (3)$$

where  $\phi_{i,j}$  is defined as the angle between the vectors  $\dot{\mathbf{x}}_i$  and  $\mathbf{x}_j - \mathbf{x}_i$ .  $f(\phi_{i,j})$  can be interpreted as the field of vision of each individual, making them sensitive to individuals directly in front and progressively less sensitive as the angle approaches 180 degrees.

This force is inspired by the Lennard-Jones (LJ) potential (graph on next page),

$$V = 4\epsilon \left( \frac{\sigma^{12}}{r^{12}} - \frac{\sigma^6}{r^6} \right) \quad (4)$$

where the first term represents repulsions and the second term attractions. The attraction and repulsion that individuals face is similar to those felt by atomic species (since group members are assumed to be point masses analogous to an ideal gas). When molecules are squeezed together, the nuclear and electronic repulsions and the rising electronic kinetic energy begin to dominate the attractive forces. The repulsions increase steeply with decreasing separation. LJ potential is a simplified approximation of the hard-sphere potential in which the potential energy rises abruptly to infinity as soon as the particles come within a separation distance  $d$ , in our case this represents the zone of repulsion. Hence, by using LJ force, both repulsive and attractive tendencies between group members can be taken into account. The constant  $\epsilon$  was set to  $-100 \text{ cm}^{-1}$  (typical for molecular interactions), and equation (3) was solved for  $\sigma$  such that the point where attraction turns to repulsion (the x-intercept) would be at  $r = 1$ , which parallels the limits of the zone of repulsion.

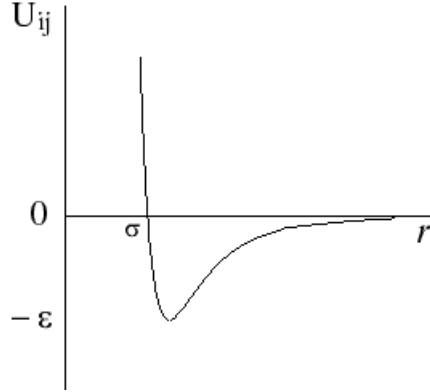


Figure 2: Lennard-Jones potential.  $\sigma$  = hard sphere diameter (=zone of repulsion),  $\epsilon$  = well depth (strength of attraction)

2. The orientation force is defined as an average acceleration:

$$\mathbf{F}_i = m_i \frac{\sum_{j \in Z_{oo}} \ddot{\mathbf{x}}_j}{N_{Z_{oo}}} \quad (5)$$

where the index  $j$  runs over all the individuals within the zone of orientation,  $m_i$  is the mass of each individual, and  $N_{Z_{oo}}$  is the total number of individuals in the zone of orientation. The justification of using such an average force is that since orientation has the physical meaning of “do what your neighbor is doing”, in terms of our force model, this is the same as taking the average of the net forces experienced by each neighbor. The resultant force is then, in the direction of the majority of the neighbors, resulting in alignment.

## 3.2 Boundary Detection

Given a function  $f(x, y, z) = 0$  and the local gradient, it is fairly simple to compute a zero of that function. Since the gradient of a scalar field, such as  $f(x, y, z) = 0$  is a vector that points in the direction of the steepest ascent, the algorithm to find the zero is to go in a negative gradient direction if  $f(x, y, z) > 0$ , otherwise go in the direction of the gradient. This method ensures finding a zero of the function, however, it may not find all the zeros. We will discuss in the Results section how group interactions can remedy this problem. In the AP framework, we define a force, an external stimulus force  $F_p$  to capture the attraction of individual  $i$  to the boundary. If  $f(\mathbf{x}_i) < 0$

$$\mathbf{F}_{p,i} = \text{grad}(f(\mathbf{x}_i)) \quad (6)$$

If  $f(\mathbf{x}_i) > 0$ .

$$\mathbf{F}_{p,i} = -\text{grad}(f(\mathbf{x}_i)) \quad (7)$$

### 3.3 Leadership

The orientation and interactive forces fully describe the social interactions among group members. However, the leaders must possess more information, namely, the location of the target ( $T$ ). This is again provided by an external stimulus force in which attraction felt by leaders to the target is an exponential function of distance from target.

$$\mathbf{F}_e = -.5e^{\|\mathbf{T}-\mathbf{x}_i\|}(\mathbf{T} - \mathbf{x}_i) \quad (8)$$

This makes physical sense, since the leaders will not feel the need to accelerate towards the target as strongly when they are within reach.

Although this AP framework nicely captures the dynamics of group behavior, several additional restrictions must be considered in order to make this model realistic. For example, animals have a blind spot, maximum turning rate (a cheetah running full speed cannot immediately turn 180 degrees) and a maximum acceleration. The blind spot has already been taken care of by using the function  $f(\phi_{ij})$ . The maximum turning radius is implemented by first finding the angle between the old and new position vectors and comparing it to a maximum turning angle (currently set at 60 degrees). If the calculated angle is above the maximum value, the new position vector is the average of the old and new position (this calculation is repeated until the calculated angle is within acceptable value).

To account for the fact that animals also have a maximum acceleration, the new acceleration for individual  $i$  at each time step is normalized by the magnitude of maximum acceleration from the group,  $(\ddot{\mathbf{x}}_i/\max(\ddot{\mathbf{X}}))$ .

Finally, with the current model, there exists the possibility that the leaders might run off to the target while the rest of the group is left far behind. This is especially likely for large group sizes since the attraction to a leader is masked by the shear large number of attraction to other members in the group. This makes it difficult for members to peel away from the group and start following a leader. To circumvent this failure, mother nature has devised a more sophisticated behavior for the leaders. In honeybees, for example, the leaders continuously fly through the swarm with their flight path aligned in the direction of the destination [1]. This way, a leader cannot get too far from the group. Thus, inspired by the honeybee model, we implemented the condition for the leaders to turn around after 40 time steps, fly back to the swarm for 10 time steps, and then go back to the target for another 40 time steps. This behaviour is repeated for the duration of the model. It is worth noting that this particular choice of numbers proves to work for large group sizes while keeping the leader size relatively small. We will investigate how realistic this condition is in future.

### 3.4 Predator Avoidance

Mathematically, avoiding a predator is the exact opposite of being attraction to a target (experienced by the leaders). Since the predator in our model is all-powerful and has no maximum turning radius, a good strategy for avoidance is to split (explode) as soon as predator threat is realized and then reform once the threat is over. More precisely, this

condition translates into: if there is a predator within the zone of predator avoidance (zop), repel from everyone around you and fly in the opposite direction of the predator. This behavior is captured by the predator avoidance force,  $F_p$  (experienced by individual  $i$ ):

$$\mathbf{F}_{p,i} = \sum_j \frac{\mathbf{x}_j - \mathbf{x}_i}{\|\mathbf{x}_j - \mathbf{x}_i\|^2} + N * \frac{\mathbf{Pred} - \mathbf{x}_i}{\|\mathbf{Pred} - \mathbf{x}_i\|^2} \quad (9)$$

where the index  $j$  runs from 1 to  $N$  and  $\mathbf{Pred}$  denotes the position vector of the predator.

If there is a predator within zop, the resultant net force on individual  $i$  is given by:

$$m\ddot{\mathbf{X}} = 0.1 * (\mathbf{F}_i + \mathbf{F}_a + \mathbf{F}_e) + 0.9 * \mathbf{F}_p \quad (10)$$

By weighing the contribution of the other social forces only 10% and the contribution from predator avoidance force 90%, the net effect will be to fly away from the predator and from each other, but at the same time, the group will not fragment too much due to the presence of  $F_a$  and  $F_i$  forces.

## 4 Results

### 4.1 Boundary Detection

The main issue is to determine if a swarm operating under interactive forces is better at outlining a boundary than a group of single individuals. A point on the boundary is said to have been found if the value of the function  $f(x, y, z) = 0$  defining a solid object is  $-0.1 < f(\mathbf{x}) < 0.1$ . A good solution to boundary detection is one in which most of the boundary space has been sampled with a fairly even distribution of detected points. This corresponds to a low ratio of density of points on the boundary to the number of total detected points. As an illustrative example, consider the problem of detecting the boundary of a sphere defined by  $f(x, y, z) = x^2 + y^2 + z^2 - 4 = 0$ . This is done by a swarm of size 100 using both local interactions (using all three forces defined) and only the external stimulus force (neglecting the orientation and interactive forces).

The figures (at the end of the report) show a relatively "good" depiction of a sphere for both scenarios; however, when the ratio of density to total dot number is compared, the solution using interactive forces is optimal (lower ratio). The solution without interactive forces gives a dot distribution localized in multiple concentric rings, which increases the density on the boundary; however, by using interactive forces, a much broader distribution of points is obtained on a boundary. This gives a good realization of the entire boundary rather than a snapshot of a local section.

The advantage of using collective group behavior becomes more important (and readily apparent) when an irregular surface is considered, such as  $f(x, y, z) = x^2 + y^2 + 20 * x^2 * y^4 + z^2 - 4 = 0$ . By comparing the actual surface, the boundary traced out by a swarm following local interactions and a swarm composed of single individuals, we can readily observe that the solution using local interactions give a much better approximation of the boundary.

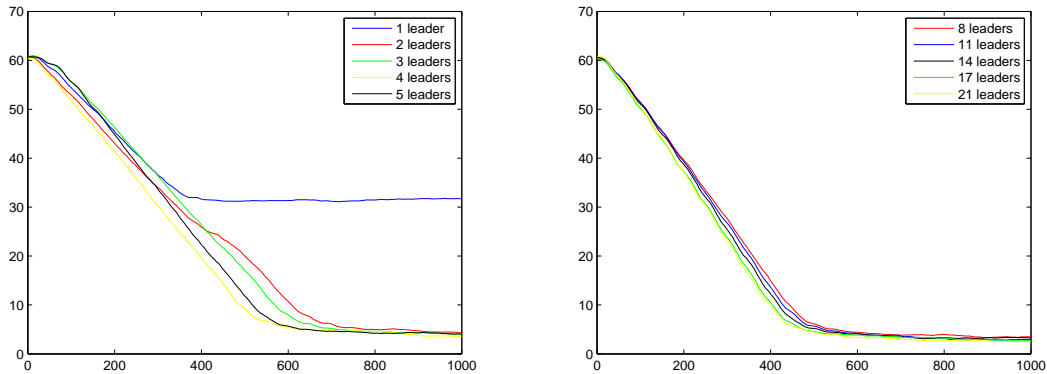


Figure 3: Distance from the center of mass to target for group size of 125 and varying leader sizes

As a final example, consider the problem of detecting the boundary of an hourglass figure, given by  $f(x, y, z) = x^2 + y^2 - z^2$ . Again, by using local interactions as well as the external stimulus force, we see that a group of size 100 can approximate the boundary pretty well. Similar experiments were done with varying group sizes and we find that performance (measured by computing density to dot ratio) is poor at smaller groups and increases drastically when group size reaches 50. It then increases gradually up to group size 100 and plateaus. This result is consistent with our previous discussion that single individuals can only solve the problem locally and miss out on the big picture. A very small group confers little interaction among group members resulting in poor performance.

## 4.2 Relation between leader size and group size

A central question of leadership is what is the number of leaders required to most efficiently guide a group to a final target. For example, for a group size of 125, at least 8 leaders are required (figure 3). This was determined by plotting the distance from the center of mass to the target (set at  $\langle -35, 35, 35 \rangle$ ) as a function of time for various leader sizes.

Our previous results strongly suggest that there is a functional relationship between the swarm size and the leader size. Thus the minimum number of leader were determined for group sizes of 50, 75, 100, 125,..., 300. For each group, 1%...20% leaders were used and the minimum number of leaders that can bring the center of mass closest to the target after 1000 time steps was determined. A plot of minimum leader size versus group size reveals a linear relationship between the two and suggests that the leader size is approximately 6% of the total group size, consistent with reported value of 5% found in the literature (figure 4). The data for small group sizes does not agree well with the LS fit and those experiments will be performed again to validate their accuracy.

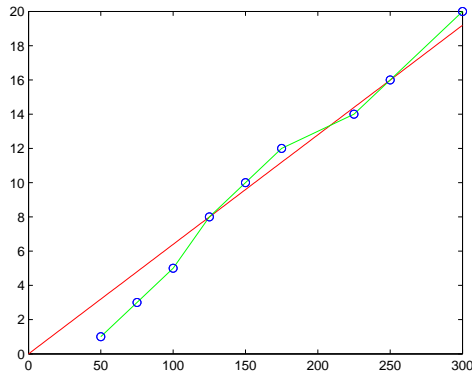


Figure 4: Relationship between leader size and group size. Linear regression yields leader size =  $0.0634 \times \text{group size}$  with an  $R^2 = 0.963$ . Blue circles = actual data, red line = LS fit, green = curve fit

### 4.3 Predator Model

The optimum strategy for avoiding predator is to scatter and then reform; in doing so, one individual is sacrificed to save the rest. Using a Coulombic repulsion force proves very useful as it provides a fluid response and mimics natural behaviour in that everyone flees from the predator and themselves in an attack. The prey flees in an opposite direction from the predator until its inevitable death, meanwhile the rest of the swarm reforms behind the predator and resumes its normal behaviour. Also, the fleeing behavior of one individual naturally propagates down the swarm (through the orientation force).

The objective of this swarm was two fold: get to a target and avoid predator. Using the predator avoidance force,  $F_p$ , it is easy to see that the group will avoid a predator. To demonstrate that the group also reaches its target (when predator is not present), we plot the time it takes to kill a tenth of the swarm in a population of 100 individuals and seven leaders. On the same figure, we also plot the distance from the center of mass to the target. These two plots are consistent in that the  $\|\mathbf{com} - \mathbf{target}\|$  peaks when the predator is near its prey and reaches a minimum after the predator is gone (figure 3).

Since the plot of  $\|\mathbf{com} - \mathbf{target}\|$  is periodic with a period corresponding to the time it takes to kill a member of the group, we need to find the period of the oscillations. In our current implementation, the zone of predator detection is set at 25. Once the swarm reaches equilibrium, that is, explode when predator reaches within distance of 25 and then reform when the threat is over, we have the following situation. We can arbitrarily set the predator position at the origin, the prey's position 25 units away from the origin and the rest of the swarm somewhere else (farther than 25 distance). This scenario corresponds to the second half of the group behavior, namely the reforming once predator threat is gone. In the end, we must multiply our result by 2 to get the actual period.

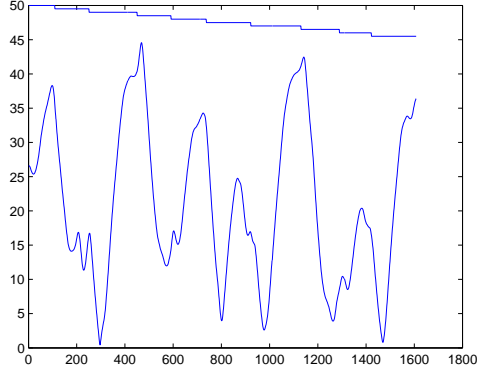


Figure 5:  $0.5 * (\text{Total group size})$  (top plot) and the distance from center of mass to the target (bottom plot) as a function time.

The predator's initial velocity is 1, acceleration 1, and a time step 0.8; the prey's initial velocity is also 1, acceleration 1, and time step 0.5. The predator and prey are separated by a distance of 25 and we seek the number of time steps required to put the predator 1 distance away from the prey (to make its kill). This is a problem in kinematics:

$$x_{prey} = 25 + v_{i,prey} * t_{prey} + a_{prey} * t_{prey}^2 / 2 \quad (11)$$

$$x_{pred} = v_{i,pred} * t_{pred} + a_{pred} * t_{pred}^2 / 2 \quad (12)$$

$$x_{prey} - x_{pred} = 25 + t * (t_{prey} - t_{pred}) + t * (t_{prey}^2 - t_{pred}^2) / 2 \quad (13)$$

where  $t_{pred} = 0.8$ ,  $t_{prey} = 0.5$  and  $t$  is the total number of time steps.

Substituting values,

$$1 = 25 + t * (-0.3) + t * (-0.39) / 2 \quad (14)$$

and solving for  $t$  yields  $t = 162$ . As mentioned above, the period is actually twice this number = 324. Thus, on average, it takes 162 time steps to kill a member of the swarm and twice this number correlates to the period of  $\|com - target\|$  function. Thus, one may vary the zone of predation or relative speeds of predator and prey to optimize for the period. Such an analysis is not presented here.

Furthermore, it seems that increasing the number of leaders does not improve the chances of survival of the group. Although this may seem contradictory since once a group fragments, there is a probability that a certain fragment will not have a leader in which case this particular fragment will not be able to return to the target. Thus, the larger the group size, the better the chances of there being a leader in a fragmented group and therefore better the overall behavior. However, leader size does not seem to matter because in our model, the leaders always go back to the swarm so that even a leader-less fragment will not be lost.

Although our model is effective in avoiding a predator and reaching the target, sometimes we observe that upon scattering, an individual fails to come back to the group, nevertheless

the number of individuals lost compared to the group is relatively small. The honeybee model plays a role in getting the swarm back to the target since the leaders do not stay at the target but keep going back to the swarm and lead them back to the target.

## 5 Conclusion

We provide a simple framework to study swarm behavior arising from a set of local interactions. Many interesting features arise naturally such as leadership, target avoidance, and boundary detection all of which have important practical usage. The group size is of course one of the most important parameters in this model, however, it does not need to be very large since, according to the swarm mentality, the sum of the parts is larger than the whole. A group of individuals working together accomplish a lot more than group of single individuals acting alone. This is especially true for boundary detection problem where only 100 individuals are required to map out most regular or irregular surfaces and for leadership where only 5% of the group needs to have prior knowledge of the destination.

## References

- [1] N. W. A. Merrifield, M.R. Myerscough. A model of spatial sorting in animal groups, with an application to honeybee swarm movement. *Elsevier Science*, 2004.
- [2] R. G. I. Couzin, J.Krause and N.Franks. Collective memory and spatial sorting in animal groups. *Journal of theoretical biology*, 218(1):1–11, 2002.
- [3] C. W. Reynolds. Steering behaviors for autonomous characters. *Sony Computer Entertainment*, 1999.
- [4] D. G. William Spears. Using artificial physics to control agents. *Proceedings of the 1999 International Conference on Information Intelligence and Systems*, pages 281–291, 1999.

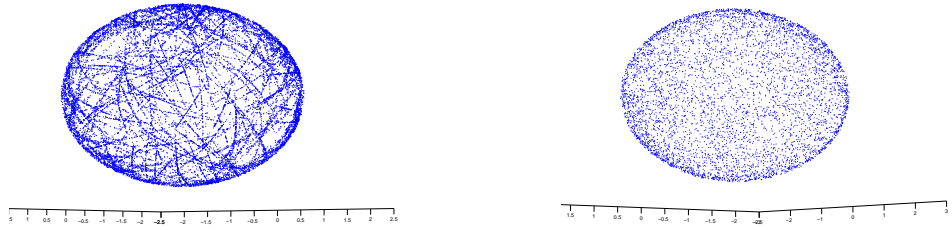


Figure 1: Boundary of a sphere detected by swarm of size 100 using only the external stimulus force (left) with a density to dot ratio = 0.010; and using interactive forces (right) with density to dot ratio = 0.0042

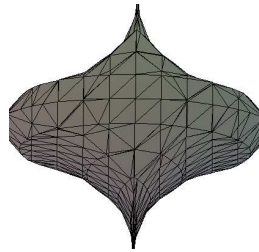


Figure 2: Surface plot of the irregular shape. Swarm solution to determine the boundary of this surface is presented in the figures below.

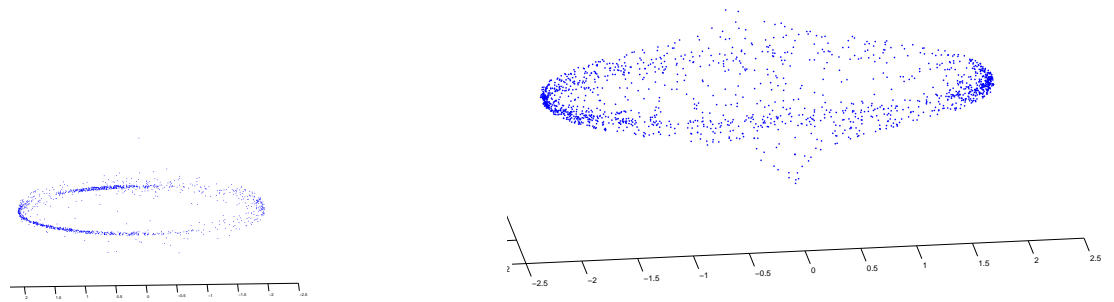


Figure 3: Boundary of an irregular surface detected by swarm of size 100 using only the external stimulus force (left) with density to dot ratio = 0.0514 and using local interactions (right) with density to dot ratio = 0.0231.

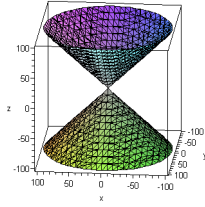


Figure 4: Actual surface of the hourglass.

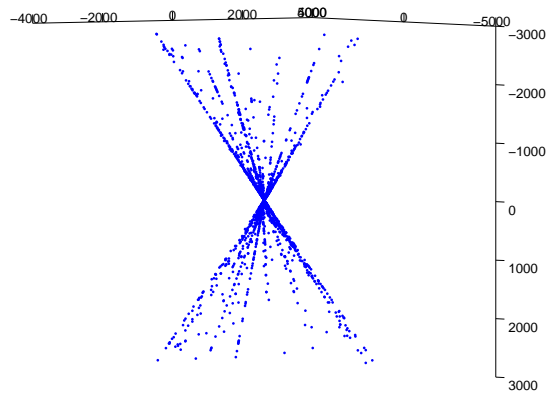


Figure 5: Boundary of an hourglass detected by swarm of size 100 using local interactions with density to dot ratio = .011.