Predicting the Fall of Dominoes
Milestone 5

Suzy Smith, Jenna Gil, Ashley Archibald

December 9, 2005

Abstract. By studying the physics behind toppling dominoes we are able to create a theory to predict the time for the dominoes to fall and if necessary, find an arrangement of dominoes that will optimize the topple time. The conservation of energy is used and all interactions between the dominoes are neglected except for the initial impact. Using the number of dominoes, the geometry of the chain, and the spacing between the dominoes we are able to calculate and optimize the topple time.

1 Introduction

Who hasn't heard of the domino effect? It begins with a single domino. As that single domino falls it comes into contact with the next domino causing that domino to topple and come into contact with the domino after that, and so the game is played until all the dominoes have fallen.

People may not realize that this seemingly simple demonstration is at the root of many complicated occurrences throughout nature. Many theories throughout ecology, psychology, and sociology, as well as many others, can all be understood and displayed through experimentation with falling dominoes. The dynamics of a domino chain can simplify the phenomenons behind nerve cell communication, ice shelves, molecule cascades, and even crop circles.

This semester we have studied the mechanics behind falling dominoes. We now have a better understanding of how the amount, spacing, and initial trigger of the dominoes can effect the progression of a domino cascade.

How quickly will a line of dominoes fall? Can a cascade of falling dominoes be stopped? How will the shape of the track affect the speed of the cascade? These questions are what we have theoretically and experimentally investigated throughout the semester.

For our final milestone, we have combined all the studies and research we have collected throughout the semester. We will calculate the topple time of a given domino chain as well as attempt to create a set up to minimize the topple time when given the total length of the chain and the number of dominoes. We will also use our theory to estimate the time for the setups created by our classmates.

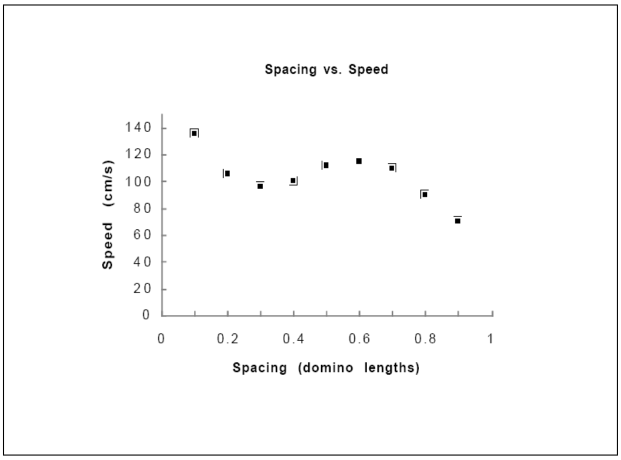
2 Existing Literature

In order to create our own theories and experiments for predicting the fall of domino cascades, we took at look at what others have concluded on the subject.

The Exploratorium [2], The Museum of Science, Art and Human Perception in San Francisco, CA has studied how the toppling of dominoes can imitate impulses being sent through nerve cells of the human body. They have drawn simple yet important conclusions from their research and experimentation. First, in order to begin the cascade, the first domino must be pushed past a critical angle. Once it begins, the 'pulse' of the falling dominoes moves at a constant speed and is independent of the force of the initial push. This pulse of falling dominoes will travel in only one direction. The Exploratorium has provided an outline on the experiments they conducted in order to come to their conclusions. This source has provided us with background information about dominoes being tested at an equal distance apart along a straight line and how those conclusions can be used as a model for the transmission of impulses through nerve cells.

'How Fast Do Dominoes Fall?' [3] looks into the issue of spacing as a function of domino toppling time. It investigates topple time as different spacings between the dominoes are used and attempts to find a maximum speed for a row of 100 dominoes. Their findings include that dominoes spaced relatively closely together (0.10 units relative to the length of one domino) will take a longer time to topple because the speed with which the dominoes fall is decreased when they are placed close together. They also found that when dominoes are placed far from each other (0.90 the length of the domino) the topple time is also low because it takes a longer amount of time for one domino to come into contact with the next. Their maximum speed is reached somewhere in the middle (0.60 the length of one domino). From this source, we could study experiments of equally spaced domino chains that were previously conducted and view their results.

From an article by Charles W. Bert [1], we see the application of the conservation of energy to a system of dominoes. The energy of the system will remain constant whether in the form of kinetic energy, $\frac{1}{2}I\omega$, or potential energy, $\frac{1}{2}Mg\sqrt{H^2+T^2}$. Therefore, by finding



the initial energy of the system, Bert was able to develop a theory to predict the topple time of a cascade of n dominoes; he is assuming a certain amount of initial kinetic energy is applied to the system to begin the progression.

Bert considers the number, geometry and spacing of the dominoes in his theory and expresses his results in dimensionless graphs. This theory is based upon a number of assumptions, i.e. identical dominoes, no slipping, no external friction losses and no contact between the dominoes other than initial impact. Bert begins by studying the fall of a single domino. Next, he experiments with multidomino chains. He determines the propagation velocity as the distance between each domino over the total time period from initial impulse to contact with the next domino. He then finds a dimensionless quantity for propagation velocity by dividing V by $\sqrt{gh}$, which is the equation for the velocity of a shallow water wave. Bert also finds dimensionless values for propagation time and normalizes the initial kinetic energy by the initial potential energy and calls it n. Bert next plots dimensionless V by n. His experimental results are based on those found by McLachlan et al and he concludes that any discrepancies between this theory and McLachlan's results come from external friction loses and the fact that his assumption that dominoes do not interact after the initial impact is false. From this source, we can study a theory that assumes that there are no interactions between the dominoes other than their initial impact.

From an article by D. E. Shaw [5] entitled 'Mechanics of a chain of dominoes', we found a computer-aided theoretical prediction for 'the time required for a linear chain of equally spaced dominoes to fall.' Like Bert, Shaw also derived his theory using the basis of conservation of energy. However, unlike Bert, Shaw suggests that once a domino strikes the next it will continue to exert a force upon the second domino. This article predicts a nearly linear relationship between the time for N dominoes to fall, t_N, and the number of dominoes, N; he suggests that this is true for cascades where $N > 6$ because the important contributing factors to energy are fairly constant at this number. He states that his theory is only adequate for experiments conducted on less than 6 dominoes. The following equations were found by Shaw:

$$\sin(\theta_{N-1} - \theta_N) = \frac{c\cos(\theta_N)}{b} - \frac{a}{b}$$

$$\omega_{N-1} = [1 - \frac{(c\sin(\omega_N))}{b\cos(\omega_{N-1} - \omega_N)}]\omega_N$$

$$E = \frac{1}{2}mg[b\Sigma_1^N(\cos\theta_i) + a\Sigma_1^N(\sin\theta_i)] + \frac{1}{2}I\Sigma_1^N(\theta_i^2)$$

From the equations in this article, we believe that Shaw was beginning the experiment with the first domino resting on the second. He released the second domino and allowed the potential energy to become kinetic. He measured the time, t, from the second domino being released until the final domino fell. His formulas do not take into account any time needed for the first domino to come into contact with the second. His equations are also only valid for a $\theta < 75\,\text{deg}$. From Shaw, we found another theory based upon the conservation of energy, and we are introduced to the idea that dominoes continue to interact with each other even after their initial contact.

Stronge and Shu [6] base their theory on the fact that there is a natural speed of propagation of a cascade of dominoes and the 'wave of destabilization' will asymptotically approach that speed as the cascade advances. They state that the natural speed increases as the number of cascade members increase. Similar to Shaw, they assume dominoes stay in contact with the next after initial impact. This theory of destabilization is fashioned for an infinitely large number of dominoes.

The equations Stronge and Shu produce as a starting point for their theory are the same as Shaw [5] derived for his. They are based on the geometry of the domino chain. Stronge and Shu go on to derive a relationship between the collision speed of a domino and its initial speed, and also a domino's initial speed and the initial speed of the adjacent domino.

$$\frac{\psi_1}{\phi_1} = [(\frac{k}{k-1})(1 - \frac{p}{kK_1})]^{\frac{1}{2}}$$

where ψ_1 is the collision speed, ϕ_1 is the initial speed, k is the energy ratio, K_1 is the kinetic energy and p is the potential energy. And,

$$\frac{\phi_0}{\phi_1} = [(\frac{k-1}{k})(1 - \frac{p}{kK_1})]^{\frac{1}{2}}$$

where ϕ_0 is the initial angular speed.

When the number of dominoes is limited, the available energy source reduces and results in the reduction of the natural speed of the cascade and an increase in the minimum spacing necessary to achieve that natural speed. Within their article, the authors state that after the 'destabilization wave' has passed the first 6-15 dominoes, the time between initial impact on the domino and when the domino collides with the next becomes fairly constant with consistent variation.

Stronge and Shu draw of a number of conclusions from their studies. First, the cascade of dominoes has a natural speed that depends upon the interactions between the dominoes. This varies from other theories where there is assumed to be no interactions between the dominoes other than the initial impact. Also, Stronge and Shu believe that a collective group of dominoes applies pressure to the domino at the wave front that is dependent on the number of dominoes. The authors state that the number of significant dominoes decreases as the spacing between them increases. They believe the natural speed depends on the rate of change of the potential energy of the group of dominoes as the cascade progresses. Stronge and Shu neglect friction and multiple collisions between the dominoes that may occur during experiments. They state that the differences between their theory and experimental results come from perturbations for the steady solution and soon dissipate.

Van Leeuwan [7] draws from Shaw's [5] research and theory that once the proceeding domino comes into contact with the next, it will remain in contact and continue to exert a force upon it. However, Van Leeuwan disagrees with Shaw's theory that angular momentum is conserved and demonstrates his argument by analyzing the forces between the dominoes.

Van Leeuwan makes all of the usual assumptions that are necessary in order to derive his equations. He assumes the dominoes are identical and perfect and do not slip on the table's surface. He states the fall of the dominoes is due to gravitational force. His goal is to find the dependence of the soliton speed on the interdistance s/h and make the collision law more precise. (s is the distance between the back of one domino to the front of the next and h is the height of one domino.) His equations are valid only for fully developed cascades, $N > 6$.

3 Assumptions and Predictions

We made a number of assumptions about our domino chains in the course of our experiments and theoretical calculations. Our assumptions are as follows:

1 All dominoes in the chain are identical. We have used the average height, width, and thickness of 8 dominoes to approximate the values for all dominoes.

2 There is no sliding between the table and the dominoes and no external friction losses. By conducting our experiments on sandpaper, we are able to limit sliding and make this assumption.

3 Within the system, energy is conserved. This is a fairly good assumption following the previous assumption neglecting friction losses.

4 Each domino rotates in one direction only. We assume all motion is moving forward. Especially with curving chains, we assume there is no sideways rotation of the dominoes.

5 Experiments were conducted on a perfectly flat surface. The table was never measured, but this is a fairly good assumption to make.

6 The coefficient of restitution is negligible. The coefficient of restitution is the measure of the elasticity of the collision between the dominoes. We assume that there is no bounce and no second impact of the dominoes on one another, therefore, we neglect the coefficient of restitution.

7 The initial angle of the first domino is very small and we assume it to be zero. We also assume that the each domino begins at rest and its initial velocity is zero.

For the time prediction of a curved arrangement, we predict that the topple time will be longer than if the dominoes were presented in a straight line for the same distance. Because we are not taking the dominoes' rotations into account, we are expecting our experimental values to be higher than our calculated topple times.

For the optimization of topple time of a domino chain, given the total length and number of dominoes, we use our previous experimentation and theoretical findings to make our prediction. We believe that the minimum topple time will be achieved through a setup that begins with the dominoes relatively far apart and ends with them being closer together.

4 Predicting the Topple Time of a Domino Chain

All of our sources for investigating the topple time of a chain of dominoes conducted their experiments on equally spaced setups of domino chains. We believe we can expand upon what we have learned from these sources and create a theory valid for varying spacing throughout the chain. We have also integrated into our theory a prediction of the topple time for dominoes in a curved path. The geometry of our toppling dominoes is given below.

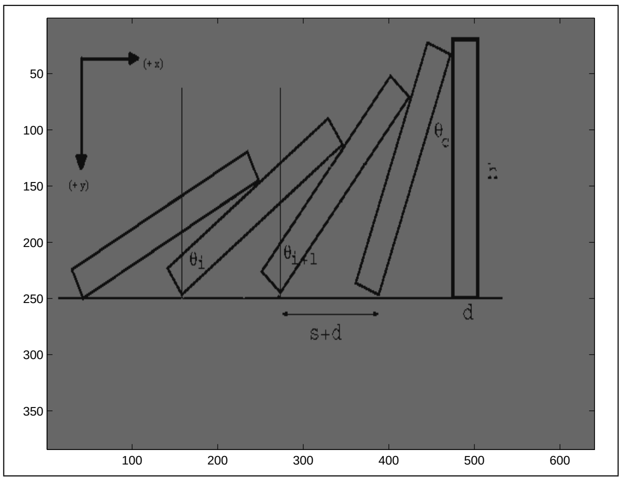


Figure 1: Geometry of a Domino Cascade

Our topple time theory is derived from the conservation of energy equation that we found using the geometry of the toppling domino chain from above and equations from Bert's [1] and Shaw's [5] publications,

$$\frac{1}{2}I(\frac{d\theta}{dt})^2 - mg\frac{D}{2}\cos\theta = \frac{1}{2}I(\frac{d\theta_0}{dt})^2 - mg\frac{D}{2}\cos\theta_0 \tag{1}$$

This equation states that the sum of the kinetic and potential energies remains constant throughout the system. Within this equation, I is the moment of inertia, $I = \frac{m}{3}(h^2 + d^2)$, m is the mass of one domino, and D is the diagonal height of the domino, $D = \sqrt{h^2 + d^2}$. The right side of the equation is the initial energy of the system, E_0, and is a constant. Because the system begins at rest, there is no initial kinetic energy and the equation becomes,

$$\frac{1}{2}I(\frac{d\theta}{dt})^2 - mg\frac{D}{2}\cos\theta = -mg\frac{D}{2}\cos\theta_0 = E_0 \tag{2}$$

We then take the derivative of both sides, canceling like terms to get,

$$I(\frac{d^2\theta}{dt^2}) + mgD\sin\theta = 0 \tag{3}$$

From this point, we non-dimensionalize our equation. Our two variables to non-dimensionalize are the angle, θ, and time, t. Since θ is already dimensionless, we only need to non-dimensionalize t. We begin by creating a dimensionless T and setting T equal to $\frac{t}{\tau}$.

$$T = \frac{t}{\tau} \tag{4}$$

We solve this equation for t and find $\frac{d\theta}{dt}$ and $\frac{d^2\theta}{dt^2}$ in terms of T.

$$\frac{d\theta}{dt} = \frac{1}{\tau}\frac{d\theta}{dT} \tag{5}$$

$$\frac{d^2\theta}{dt^2} = \frac{1}{\tau^2}\frac{d^2\theta}{dT^2} \tag{6}$$

We plug these values back into equation (3) and manipulate to get,

$$I\frac{1}{\tau^2}(\frac{d^2\theta}{dT^2}) = -mgD\sin\theta \tag{7}$$

Solving for $\frac{d^2\theta}{dT^2}$ yields,

$$\frac{d^2\theta}{dT^2} = -\frac{mgD\tau^2}{I}\sin\theta \tag{8}$$

In order to simplify the equation we set the constant value $\frac{mgD\tau^2}{I}$ equal to 1, which means τ must have the value,

$$\tau = \sqrt{\frac{I}{mgD}} \tag{9}$$

With this simplification, we are left with,

$$\frac{d^2\theta}{dT^2} + \sin\theta = 0 \tag{10}$$

This equation can be rewritten as,

$$\frac{d}{dT}\left(\frac{(\frac{d\theta}{dT})^2}{2} - \cos\theta\right) = 0 \tag{11}$$

Integrating we find,

$$\frac{(\frac{d\theta}{dT})^2}{2} - \cos\theta = Constant \tag{12}$$

From initial conditions, $\theta_0 = 0$ and $\frac{d\theta}{dt} = 0$, we find this constant to be equal to the initial energy, E_0. Solving for $\frac{d\theta}{dt}$ we get,

$$\frac{d\theta}{dT} = \sqrt{2(E_0 + \cos\theta)} \tag{13}$$

Separating the variables and integrating we find T_i.

$$\int_{\theta_0}^{\theta_c} \frac{d\theta}{\sqrt{(2(E_0 + \cos\theta))}} = T_i \tag{14}$$

Converting dimensionless T back into t (s),

$$\int_{\theta_0}^{\theta_c} \frac{d\theta}{\sqrt{(2(E_0 + \cos\theta))}}\sqrt{\frac{I}{mgD}} = t_i \tag{15}$$

In the above equation t_i is defined as the time between when the domino first begins its forward motion and the time it comes into contact with the next domino or the table. The critical angle, denoted θ_c, is the angle at which the domino comes into contact with the next domino. From the geometry of our falling domino chain, we define the critical angle as,

$$\theta_c = \arcsin\frac{s}{h} \tag{16}$$

When the path of the domino is curved, θ_c is effected because of changes to s. s is the horizontal distance from a point along the top of the first domino in its initial position, to the point of contact of that domino with the next. This distance is found to be

$$s = l\tan\theta_r - d \tag{17}$$

In this equation, l is the length from the edge of the domino to the pivot point, θ_r is the angle between the dominoes, and d is the thickness of one domino. This relationship is derived from the geometry of the system.

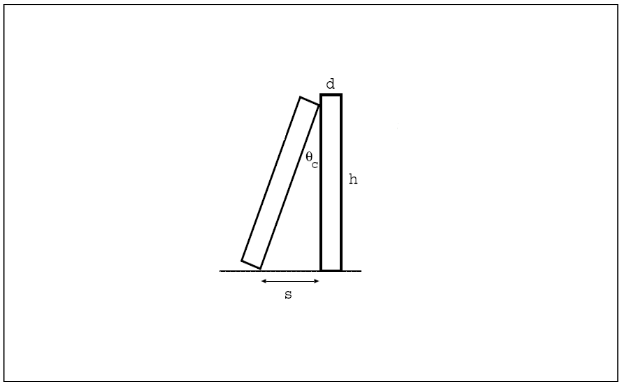


Figure 2: Geometry of the Critical Angle

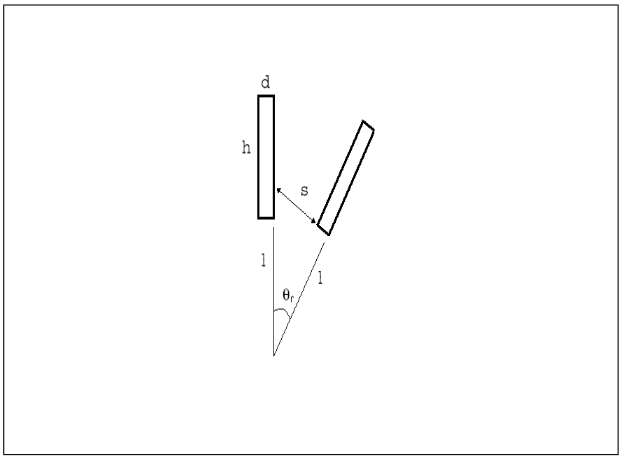


Figure 3: Geometry of a Curved Domino Cascade (Top View)

For the last domino of the cascade, the critical angle will be equal to $\frac{\Pi}{2}$ whether the path is curved or straight.

By taking a sum of all the t_i we can calculate the total topple time, t_N for a cascade of N dominos.

$$\int_{\theta_0}^{\theta_c} \frac{d\theta}{\sqrt{(2(E_0 + \cos\theta))}} \sqrt{\frac{I}{mgD}} = t_i \tag{18}$$

$$t_N = \Sigma_1^N t_i \tag{19}$$

5 Optimization of Topple Time

With our theory, we created a way to predict a setup that will produce the optimal topple time for a chain of falling dominoes. When we are given a total length for the domino chain, we can attain the necessary spacing between dominoes to minimize the time for all the dominoes to fall.

We started with a given length. We used the lengths that were measured in our experiments so that our theory would be easily comparable to our experimental data.

With our given length, we developed a spreadsheet in Microsoft Excel that lists the given values needed in our theory (i.e. Moment of inertia, mass, height and thickness of a domino, initial energy, etc.). The spreadsheet also lists a value of spacing, s, ranging from 1.4 cm to 4.4 cm. We chose 1.4 cm as our lower limit because we found through experimentation that dominoes will not fall if they are spaced too closely together. We chose 4.4 cm as our upper limit because that is the average height of the dominoes we measure and if they were spaced further apart, the first domino will not come into contact with the next when toppled.

We varied the spacing down the list by 0.2 cm. Next, we calculate the time it takes for two dominoes spaced at the set distances to come into contact when the first is released from an initial angle that we assume to be zero.

After we calculated the time for each spacing of dominoes and recorded that data in our spreadsheet, we found the number of dominoes needed for each spacing within our given length. We found this number using:

$$N = \frac{(L-d)}{(s+d)} \tag{20}$$

and rounding this number to the nearest integer. Because the given length includes the first and last domino thicknesses, we subtract d from our overall length, L, in the numerator. The denominator is $(s+d)$ because that is the spacing from the front of one domino to the front of the next. By knowing the time for one domino to fall and the total number of dominoes, we were able to calculate the total topple time, t_N, for each spacing.

$$t_N = (N-1)t + 0.176937 \tag{21}$$

Because all of the dominoes in the chain have the same spacing except for the last domino, we can multiply one less than the total number of dominoes by the time calculated for that spacing and then add the time for the last domino to fall to find the total topple time for the equally spaced chain. The final domino's topple time was calculated through $\frac{\pi}{2}$ radians.

After the total time for each spacing was calculated, we created a graph to show the relationship between spacing and topple time within a given length.

The x axis is labeled with spacing, s. Because the ranges of spacing were very small and varied consistently, we were able to use the value of s on our plot. The y axis is label with the total topple time of the system of dominoes, t_N. With these labels, we have created a graph of the total topple time versus the spacing.

We had originally set out to create a graph with 2 dimensionless parameters to compare all of the variables we need to observe, but found as we proceeded that it was not necessary and we could view the domino chain's behavior with a simple plot of t_N vs. s.

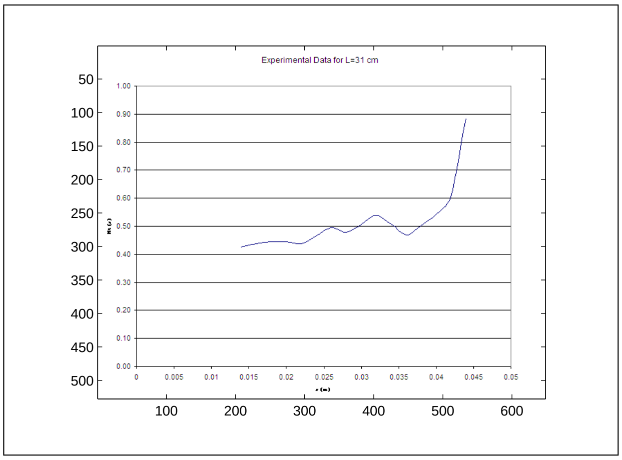


Figure 4: t_N vs. s for $L = 0.246m$

From our graph, we found that equal spacing produces the minimum topple time at a little over 2 cm. We also see that after this spacing, the topple time increases and then again decreases to a minimum at 3.5 cm.

Within our calculations, we did not take into account the manipulation of the last spacing in each chain. Because of this, we will see errors within our theoretical results.

6 Experimentation

Introduction

Our experiments are conducted to help us test our theory and give us a better understanding of the dynamics of a domino cascade. Using the Fastcam 512PCI, we monitor and observe the dominoes as they topple.

Throughout the semester, we have conducted experiments on a single domino, chains of dominoes with varying length, spacing and orientation. The following is a summary of our experimental findings.

Procedure

At random we selected 12 dominoes to use during our experimentation. Eight of the dominoes were measured for height, h, thickness, d, and mass, m, and an average value was calculated for each quantity. We decided to use an average value because the competitions would be conducted with random dominoes. For a random sample, an average m, h, and d of many dominoes is a relatively good prediction.

We conducted our experiments on sheets of sand paper to reduce the sliding between the domino and tabletop. For this reason, we assume that friction is neglected during our experiments and theoretical calculations.

We measured the distance, (s+d), from the front of one domino to the front of the next domino with a ruler. In cases of curved pathes we calculated the spacing between dominoes through the geometry of the setup. The initial angle was small enough to be assumed zero for our purposes. We used the Fastcam 512PCI to measure the total topple time of the cascade.

We began our experimentation with the study of one domino. Next, we experimented with equally spaced domino chains. We moved onto varied spacing between the dominoes and eventually domino chains with curved paths. Our first series of experiments were conducted simply to record topple time; the final experiments were performed to find the setup to produce a minimum topple time.

Results

Our first experiments were conducted upon a single domino. We ran the experiment 3 times with the same domino and the average of our experimental values is listed below.

N Dominoes	t_N
1	0.2021

Table 1: Average Topple Time for a Single Domino

Our next series of experiments were conducted using equally spaced domino chains. We varied the number of dominoes and the spacing for each experiment and ran each experiment only once. Our initial lab data revealed that the Fastcam 512PCI produced fairly consistent results and by reducing the number of trials, we were able to conduct more experiments. The results of our equally spaced domino experiments are listed below.

N Dominoes	$(s+d)(cm)$	$t_N(s)$
5	2	0.4159
6	2	0.4706
6	2.5	0.490
6	3	0.4840
6	4	0.676
8	2	0.4569
8	2.5	0.471
9	2	0.486
9	2.5	0.504
9	3	0.641
10	2	0.492
10	2.5	0.530

Table 2: Topple Times of Equally Spaced Domino Chains

To see the relationship between the spacing and the topple time, we graphed our results when 6 equally spaced dominoes were used.

Our next experiments measured the topple times of domino arrangements when the spacing was varied throughout the chain. Table 3 lists the results from our varied spaced series of experiments.

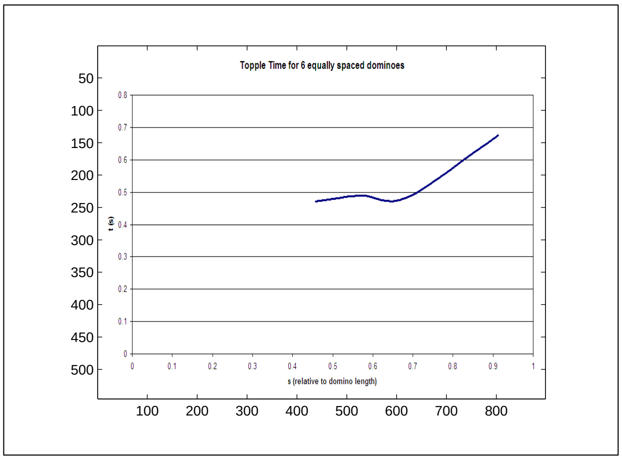


Figure 5: Topple Time for Equally Spaced Dominoes

N Dominoes	5	5	8	8
$(s+d)_1$	2	3.5	2	3.5
$(s+d)_2$	2.5	3	2.5	2.5
$(s+d)_3$	3	2.5	2.5	3
$(s+d)_4$	3.5	2	3	2.5
$(s+d)_5$	–	–	3	2
$(s+d)_6$	–	–	3.5	3.5
$(s+d)_7$	–	–	3.5	2
$t_N(s)$	0.4409	0.4170	0.5630	0.5690

Table 3: Topple Times of Varyingly Spaced Domino Chains

We also conducted several experiments using a curved domino path. Our experiments were limited to equal spacing between the dominoes. Because the results had a large deviation, we conducted 4 trials of each experiment and gained an average. The average values are displayed below.

N Dominoes	$\theta(deg)$	$l(cm)$	$t(s)$
5	20	7.75	0.4588
3	45	5	0.3870

Table 4: Topple Times of Curved Domino Paths

Our next experiments were conducted to determine the minimum topple time for a chain of domino setup when the overall length is given. We began this set of experiments by picking a random length of the domino chain. In our first series of experiments, this value was 31 cm. Our first experiments were conducted using equal spacing between the dominoes to find the optimal spacing for the minimal time. The final domino had to be manipulated so the domino chain would fill the entire required length. We recorded the spacing of the dominoes and the number of dominoes used in each experiment. Once we had several data points for equal spacing, we plotted the points on a t vs. s graph to find the optimum spacing.

Next, we began to vary the spacing between the dominoes. We kept our total chain length as 31 cm to ensure comparable results. In our first experiment with varied spacing, we began the chain with shorter spacing (relative to the domino length) between dominoes and gradually increase the spacing as the chain progressed. For our next experiment, we began the chain with a larger spacing and gradually decreased the space between dominoes. We conducted several experiments using these setups, varying the spacing each time. We recorded the spacing between each domino, the number of dominoes used, and the total time the chain took to topple. Next we tried alternate spacing between the dominoes. The first two dominoes were spaced 2 cm apart, followed by a spacing of 3.5 cm, followed by a 2 cm space, and so on until the required distance was reached. We experimented with several different setups of this nature and recorded our results.

We then repeated our experiments, this time choosing a given length of 24cm. The results are listed below.

Our first experiments were conducted using equal spacing between our dominoes.

Given L (cm)	$N Dominoes$	$(s+d)(cm)$	$t_N(s)$
31	16	2	0.7401
31	11	3	0.7679
24.6	13	2	0.6000
24.6	11	2.5	0.6300
24.6	9	3	0.6360
24.6	8	3.5	0.6199

Table 5: Topple Times for Equally Spaced Domino Chains

The results in the table above were not manipulated to account for the unequal spacing of the final domino. We did not record the final spacing taking our results to be a fairly good estimate without it. As a result the data is slightly skewed and not taken to be exact, simply a fairly good approximation.

As can be seen through our results, 2 cm spacing gave the fastest topple times. A graph of our data reveals topple time decrease at 2 cm and increases as the spacing increase. However, once the spacing increases further (to 3.5 cm) the topple time once again decreases because there are less dominoes in the chain to topple.

Our next experiments consisted of dominoes chains with varied spacing. The results of those experiments are listed.

From these experiments, we see that domino chains that begin with larger spacing (3.5cm) and gradually decrease the spacing (to 2 cm) as the chain progresses produce the smallest topple times. Those chains with alternate spacing, beginning with the smallest distance first, also revealed some of the smallest topple times.

Lab Conclusions

We made a prediction before we began our experimental work that curved paths would produce longer topple times than parallel chains when a similar spacing was employed between the dominoes. From our experimentation, we see that this hypothesis was correct.

Experiment	1	2	3	4	5	6	7
Given L (cm)	31	31	31	31	24.6	24.6	24.6
N Dominoes	12	12	12	12	12	9	10
s_1 (cm)	2	3.5	2	3.5	3	3.5	2
s_2	2	3.5	3.5	2	3	3.5	3.5
s_3	2	3.5	2	3.5	3	3.5	2
s_4	2	3.5	3.5	2	3	3	3.5
s_5	2.5	3.5	2	3.5	2.5	3	2
s_6	3	3	3.5	2	2.5	2.5	3.5
s_7	3	3	2	3.5	2.5	2.5	2
s_8	3	2	3.5	2	2	2	3.5
s_9	3.5	2	2	3.5	2	2	2
s_{10}	3.5	2	3.5	2	2	–	-
s_{11}	3.5	2	2	3.5	2	–	-
$t_N(s)$	0.880	0.732	0.749	0.823	0.5429	0.5339	0.5589

Table 6: Topple Times for Varyingly Spaced Domino Chains

Another initial hypothesis was that when optimizing equal spacing between the dominoes, we could achieve the minimal topple time. From the experiments conducted using equal spacing, we see that the optimal spacing falls around 2 cm (approximately half the length of a domino).

When our first set of minimal topple time experiments are compared with next, we find that there are a number of setups that yield topple times that are faster than those obtained through equal spacing. Alternate spacing yielded a relatively low topple time. However, our fastest topple times were obtained when the spacing began relatively large and decreased as the chain progressed.

7 Comparison of Theory and Experimentation

Finally we compare our theoretical calculations of topple time with those that we found experimentally and calculate the percentage of error for each setup. These comparisons can be viewed in the tables below. First we compare the experimental times for the fall of one domino with our calculated topple time.

N Dominoes	t_N	t_{theory}	$\%error$
1	0.2021	0.1769	14.2

Table 7: Single Domino Data

The next table compares the experimental and theoretical times of equally spaced domino chains.

The following table is a comparison between our theoretical predictions and our actual experimental results for domino chains of varied spacing.

N Dominoes	$(s+d)(cm)$	$t_{exp}(s)$	$t_{theory}(s)$	$\%error$
5	2	0.4159	0.3106	25.3
6	2	0.4706	0.3440	26.9
6	2.5	0.3960	0.3937	0.5
6	3	0.4840	0.4514	6.7
8	2	0.4569	0.4108	10.1
8	2.5	0.471	0.4804	2.0
9	2	0.486	0.4442	8.6
9	2.5	0.504	0.5238	3.9
9	3	0.641	0.6161	3.9
10	2	0.492	0.4776	2.9
10	2.5	0.530	0.5671	7.0

Table 8: Experimental vs. Theoretical Topple Time for Equal Spacing

N Dominoes	5	5	8	8
$(s+d)_1$	2	3.5	2	3.5
$(s+d)_2$	2.5	3	2.5	2.5
$(s+d)_3$	3	2.5	2.5	3
$(s+d)_4$	3.5	2	3	2.5
$(s+d)_5$	–	–	3	2
$(s+d)_6$	–	–	3.5	3.5
$(s+d)_7$	–	–	3.5	2
t_{exp} (s)	0.4409	0.4170	0.5630	0.5690
$t_{theory}(s)$	0.3780	0.3780	0.5456	0.5242
$\%error$	14.2	9.3	3.1	7.9

Table 9: Experimental vs. Theoretical Topple Time for Varied Spacing

A qualitative comparison between the experimental topple times of straight paths and those of curved paths, show that when a path curves, the dominoes take longer to fall. Within our theory, the curvature should not play any role in the topple time, but we see from our experiments that the dominoes will twist and rotate as they fall, causing the domino cascade to proceed more slowly than it would in a straight path.

We also use our experimental data to find the setup that yield the fastest topple time for a given length and number of dominoes. The information revealed from our experiments on equally spaced dominoes reflect what we found by creating our graph of theoretical values. Equal spacing yields a minimum topple time at 2 cm and decreases again around 3.5 cm spacing.

From our experiments we found that by beginning a domino chain with 3.5 cm spacing and decreasing the spacing to 2 cm at the end of the chain, yields an overall minimum topple time.

There are inevitable variations between our theoretical model and experimental data. This due to experimental errors such as: measurement errors, flawed assumptions and the neglect of very small numerical factors such as air drag and friction. In the case of curved paths, we observed in the laboratory that the dominoes do, in fact, slide and rotate, something that was not taken into account within our theory. We also made several other assumptions when calculating our theory that could effect the percentage of error within our milestone.

8 References

[1] Bert, Charles W. (1986), 'Falling Dominoes.' SIAM Review. 28(2): 219-223.

[2] Center of Excellence for Science and Mathematics Education at The University of Tennessee at Martin. 'How Fast do Dominoes Fall?' http://www.utm.edu/departments/ed/cece/cesme/PS.

[3]Exploratorium: The Museum of Science, Art and Human Perception. 'The Domino Effect.' 2000. http://www.exploratorium.edu/ti/humanbody/dominoeffect.html

[4] Freund, David. Photo: 'Dominoes 2'. 2003. http://www.istockphoto.com (Beginning Photo)

[5] Shaw, D. E. (1978), 'Mechanics of a chain of dominoes.' American Journal of Physics. 46(6): 640-642.

[6] Stronge, W. J. and D. Shu (1988). 'The domino effect: successive destabilization by cooperative neighbours.' Proceedings of The Royal Society. 418: 155-163.

[7] Van Leeuwen, J. M. J. (2004). 'The Domino Effect.' http://www.lorentz.leidenuniv.nl/ jmjvanl/domino.pdf