# Methods of Analytic Continuation in Inverse Problems

**Abstract**: The problem of continuation (**CP**) of solutions to elliptic partial differential equations (**EPDE**) dates back to H. Schwartz, G. Herglotz and H. Poincareé, and can be stated as follows. Let $u(x)$ be a solution of a homogeneous **EPDE** with real, analytic coefficients in a domain $D \subset \mathbb{R}^n$. The problem is *to continue* $u(x)$ *across the boundary of* $D$ *as a solution of the equation as far as possible.* Note that the continuation, if it exists, is unique. Furthermore, it can have singularities in the complement $\mathbb{R}^n \setminus D$ of $D$, leading to a multi-valued analytic function of the real variable $x \in \mathbb{R}^n$.

In this talk we discuss some mathematical and physical problems related to **CP**, starting with the *Schwarz reflection principle* for harmonic functions, that can be stated as follows.

Let $\Gamma \subset \mathbb{R}^2$ be a non-singular real analytic curve and $P' \in \Gamma$. Then, there exists a neighborhood $U$ of $P'$ and an anti-conformal mapping $R : U \to U$ which is the identity on $\Gamma$, permutes the components $U_1, U_2$ of $U \setminus \Gamma$ and relative to which any harmonic function $u(x)$ defined near $\Gamma$ and vanishing on $\Gamma$ is odd; i.e., $u(Q) = -u(R(Q))$ for any point $Q$ sufficiently close to $\Gamma$. Note that if the point $Q \in U_1$, then the "reflected" point $R(Q) \in U_2$.

We consider generalizations of this reflection principle for solutions of homogeneous **EPDE**s with real, analytic coefficients and constant leading coefficients, including solutions of the Helmholtz equation.

Then we apply this reflection formula as a tool for the *continuation of boundary value problems* where the domain $D \subset \mathbb{R}^2$ has a piecewise-analytic boundary.

Another classical statement of **CP** is the so called "balayage inwards" problem. As an example of this kind of problem consider the following: *Given a body* $D$ *with a known mass distribution. Find a smaller body* $D_1$ *generating the same gravitational field outside* $D$. Note that this problem can be restated in terms of electrostatic charges instead of masses. Both statements are equivalent to the **CP** of harmonic functions. A related problem is the problem of size optimization of antennas in radiophysics with a given current distribution in some domain $D$. To construct an antenna of smaller size means that we must find a current distribution in a smaller domain $D_1 \subset D$ which generates the same electromagnetic field outside $D$. The latter problem can be reduced to the **CP** of solutions to the Helmholtz equation (or more generally to Maxwell's equations). For all the problems mentioned above the minimal size of the constructed object is determined by the singularities of the continuation of the solution to the corresponding equation. We consider an algorithm for constructing such a minimal object for the case of a gravitational field.