Course Outline
Asymptotic and Perturbation Methods - Spring 2003
Math 810
Prof. John A. Pelesko

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Office Hours: Monday 1-3, Wednesday 1-3. Office hours will be held in the MEC Lab, Ewing Room 101a.

Course web page: http://www.math.udel.edu/ pelesko/Teaching/teachingmath810.htm

Required text:

> **Advanced Mathematical Methods for Scientists and Engineers**, by C.M. Bender and S.A. Orszag

Recommended supplementary texts:

> **Multiple Scale and Singular Perturbation Methods**, by J. Kevorkian and J.D. Cole
>
> **Topics in Ordinary Differential Equations**, by W.D. Lakin and D.A. Sanchez
>
> **Introduction to Perturbation Methods**, by M.H. Holmes
>
> **Perturbation Methods**, by E.J. Hinch

Following is our course outline for Math 810 - Asymptotic and Perturbation methods. The prerequisite for this course is Math 617. Students are expected to be conversant with ordinary differential equations and to be familiar with partial differential equations. In particular, you should feel comfortable with initial and boundary value problems for ordinary differential equations as well as the classic partial differential equations of mathematical physics, i.e., the heat equation, Laplace's equation, and the wave equation. You should also be comfortable with basic techniques of scientific computing. For example, you should be able to solve an initial value problem using Matlab. In this course you will learn asymptotic and perturbation methods and how to apply them to the analysis of physical systems. Grading in this course will be based on three factors - homework, a midterm exam, and a group project. There will be approximately $5-6$ homework assignments during the semester. These will count for 30% of your final grade. The midterm exam will count for 30% of your final grade. Finally, in place of a traditional final exam, you will be required to participate in a semester long group project. This project will challenge you to not only master asymptotic and perturbation methods, but to learn how to apply them in real-world applications. Your project grade will count for 40% of your final grade.

Introduction - February 10th, 12th *An introductory problem, scaling and dimensional analysis.*

Mathematical Preliminaries - February 17th, 19th *Definition of big-O and little-o notation, definition of asymptotic expansion, integration by parts, root finding, pitfalls.*

Regular Perturbation Expansions - February 24th, 26th *Projectile problem revisited, simple nonlinear ordinary differential equations, the notion of a uniform expansion, domain perturbations for partial differential equations.*

Singular Perturbation Theory - March 3rd, 5th *More root finding, Careful analysis of an introductory problem, matching*

Boundary Layer Theory - March 10th, 12th *A complete discussion of* $\epsilon u'' + au' + bu = 0$ *with* $u(0) = A$, $u(1) = B$ *for constant coefficients.*

Boundary Layer Theory II - March 17th, 19th *Variable coefficients, interior layers, multiple layers, corner layers, nonlinear problems.*

Multiple Scale Methods - March 24th, 26th *The mass-spring oscillator with small damping. New ideas, generalized asymptotic expansion and two-timing. Midterm exam handed out this week!*

Multiple Scale Methods II - April 7th, 9th *Nonlinear oscillators, the Rayleigh oscillator, forced oscillations.*

Multiple Scale Methods III - April 14th, 16th *Parametrically forced oscillators, nonlinear waves, nonlinear diffusion.*

The WKB Method - April 21st, 23rd *Introduction to WKB theory, turning points, connection formulas, quantum tunnelling.*

Focus on Applications - April 28th, 30th *A hard boundary layer problem from the study of microsystems.*

Focus on Applications II - May 5th, 7th *The half-wave rectifier.*

Asymptotic Expansion of Integrals - May 12th, 14th *Laplace's method and the integral from our half-wave rectifier. Final project report draft due this week!*

Asymptotic Expansion of Integrals II - May 19th, 21st *Stationary phase and steepest descents.*

Exam Week *Final project reports due on day of scheduled final exam.*