

Assignment 2
Math 810 - Spring 2003
Prof. John A. Pelesko

(1) Find the leading order term in the expansion of $\lambda(\alpha)$ as $\alpha \rightarrow 0$ where λ is implicitly defined by

$$\lambda e^{\lambda^2} \operatorname{erf}(\lambda) = \frac{\alpha}{\sqrt{\pi}}$$

Using Newton's method, solve numerically for λ when α is 0.001, 0.01 and 0.1. Compare with your asymptotic result.

(2) Find a two-term asymptotic expansion of each of the roots of

$$x^2 - 2x + (1 - \epsilon^2)^{25} = 0$$

for $\epsilon \ll 1$.

(3) Find a two-term asymptotic expansion of each of the roots of

$$\epsilon^2 x^3 - x + \epsilon = 0$$

for $\epsilon \ll 1$.

(4) Find a two-term asymptotic expansion of the solution of the ode

$$y'' + y + y^3 = 0$$
$$y(0) = 0, \quad y(\pi/2) = \epsilon$$

where $\epsilon \ll 1$.