Midterm Exam
Math 810 - Spring 2003
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Following is the midterm exam for Math 810, Spring 2003. This exam is a take home exam. It is due on April 9th. You may use your books, notes, or any library resources you wish. You may discuss the problems on this exam with other students *currently* taking Math 810. You may not discuss this exam with anyone else. Of course, all work you hand in should be your own. While I will be happy to talk about the methods we have discussed in class, I will be reluctant to answer direct questions about the exam. Of course, if there are typographical errors or procedural questions, I'll be happy to discuss those.

(1) (15 points) Consider the equation

$$\lambda e^x = x$$

Characterize the number of solutions of this equation viewing x as a function of λ. For $\lambda \ll 1$ you should find that there are two solutions. Develop an asymptotic approximation to both of these solutions. Verify the accuracy of your asymptotic approximation by comparing with a numerical root finder.

(2) (15 points) Consider

$$\epsilon y'' + y' + y = 0$$
$$y(0) = e \quad y(1) = 1$$

Assume $\epsilon \ll 1$. Find a uniformly valid approximation to the solution correct to $O(\epsilon^3)$. Sketch your solution.

(3) (20 points) Consider

$$\epsilon u'' + 2xu' = 0$$
$$u(-1) = -1 \quad u(1) = 1$$

with $\epsilon \ll 1$. Find a uniformly valid approximation to the solution correct to $O(\epsilon^2)$. Sketch your solution.

(4) (20 points) Consider

$$\epsilon x'' + e^x x' - \left(\frac{\pi}{2}\sin\left(\frac{\pi t}{2}\right)\right)e^{2x} = 0$$
$$x(0) = 0 \quad x(1) = 0$$

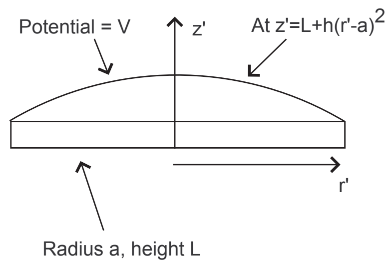

Figure 1: Sketch for problem 5.

with $\epsilon \ll 1$. Find a uniformly valid approximation to the solution correct to $O(\epsilon^2)$. Sketch your solution.

(5) (30 points) Consider a short, fat, cylinder covered by a parabolic cap. With the coordinate system shown in the figure, the bottom of the cylinder is at $z' = 0$ and is flat. The top of the cylinder is located at $z' = L + h(r' - a)^2$. The cylinder has radius a and the height of the sidewalls is L. Assume the electrostatic potential of the cap is V and the electrostatic potential of the bottom and sidewalls is zero. Setup the governing equations for the electrostatic potential inside the cylinder. Scale your equations assuming the cylinder is short and fat. Assume $h/a = O(1)$. Develop an asymptotic theory to compute an approximate solution to your system.