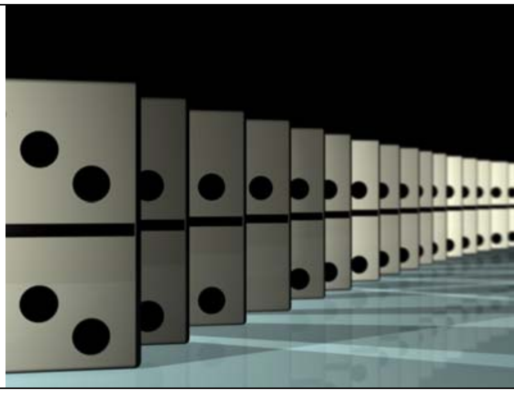
# Falling Dominoes



Quick Fact

The current world record for the largest domino cascade is held by Weijers Domino Productions. On 12/11/04, they set up and toppled a chain of 3,392,997 dominoes!

## Overview

While you are probably familiar with dominoes as a toy, you may not realize that the study of domino cascades has applications in glaciology, (Perkins 2002; MacAyeal, Scambos et al. 2003), in ecology, (Lin, Hulting et al. 2004), in power transmission networks, (Sachtjen, Carreras et al. 2000), in sociology, (Watts 2002), and even in nanotechnology, (Heinrich, Lutz et al. 2002). In all of these systems, similar questions arise. What is needed to trigger a cascade? How fast does the cascade progress? Can it be stopped? Not surprisingly, these are difficult questions, even in the case of toy dominoes. In this project, you will investigate these questions and others, experimentally and theoretically, for a domino cascade.

## Basic Questions

The central question of this project may be stated as follows: Imagine you are given n dominoes and a straight line of length L. Suppose you must stand one domino at the left end of the line and one domino at the right. How should you place your remaining dominoes such that if the domino on the left is toppled, then the domino on the right falls as quickly as possible thereafter? Associated with this central question are numerous questions that must be answered, or can be viewed as extensions. For example:

1. If you are given an arrangement of dominoes and one is toppled, can you predict how long it will take for the entire cascade to proceed?

2. How does "topple time" vary as a function of domino spacing? Domino size? The initial disturbance?
3. Suppose the dominoes are arranged on a curved track. How does topple time vary as the shape of the track varies?

It is expected that you will uncover, define, and investigate questions like these during your work on this project. Some directions will be suggested by the challenges below, while others will arise from your own interests.

## Challenges

This semester, several teams will be working on the domino project. In addition to your project report and associated milestones, there will be a series of competitions between your team and the other domino teams. The four competitions are outlined below.

1. **Week of October 3** - The first challenge is a predictive challenge. I will provide each team with a linear domino arrangement. For this competition, I will limit the arrangements to equally spaced dominoes. Prior to any experiments, your team must predict the topple time. A series of experiments will be performed and your prediction compared with the mean topple time. Teams will be ranked and scored according to the accuracy of their predictions.
2. **Week of October 24** - In the second challenge, each team will be given as many dominoes as they wish and a fixed distance, L, will be designated. Your goal is to construct a domino setup that minimizes topple time. Teams will be ranked according to their total topple times.
3. **Week of November 14 -** In this challenge, you will construct a domino arrangement for another team and challenge them to predict topple time. Your team must have a *theoretical* prediction of the topple time for any arrangement you propose. Each team will be scored according as how well they meet their challenge relative to the challenging team's theoretical prediction. The challenging team will be scored on this basis as well. Note, proposing an arrangement for which you have a poor prediction and an opposing team has a good prediction opens you up to scoring poorly twice!
4. **Week of December 5 -** The final challenge of the semester is based on the central question of this project. You will be given a distance L, n dominoes, and 30 minutes to set up a linear arrangement to minimize topple time. I will initiate all cascades in the same way. Teams will be ranked according to the topple time of their setup. The

second part of this challenge is predictive. You will be asked to predict the topple time for the other teams arrangements. Teams will be ranked according as the accuracy of their predictions. The third part of this challenge is also predictive. However, this time, I will provide an arrangement, not necessarily linear, and you will be asked to predict topple time. Again, teams will be ranked according as the accuracy of their predictions.

## Experiments

Experiments with toppling dominoes are relatively easy to perform. You have full access to the MEC Lab and associated resources to perform any experiments with domino cascades that you wish. Of particular use will be the dominoes you will find in the lab, photogates and associated software, and our video, still, and high speed cameras. As you read the literature on the domino problem you should keep in mind experiments that others have carried out and new experiments that are suggested. You will undoubtedly find that high speed photography is necessary for understanding domino motion during a cascade.

## The Literature

Several investigations of domino cascades can be found in the literature. Two very accessible references are (Banks 1998; Wagon, Pontarelli et al. 2005). Earlier references include (Shaw 1978; Bert 1986; Stronge 1987; Stronge and Shu 1988; McGeer and Palmer 1989). In reading these references you will find that there is still disagreement over what is needed in a model of domino toppling. You will also find that the state of the art in modeling domino cascades is limited to linear arrangements of equally spaced dominoes. Hopefully you can do better!

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