

**Assignment 5**  
**Math 503 - Fall 2006**  
**Prof. J. A. Pelesko**  
**Due October 16, 2006**

(1) (15 points) Search the internet and find an example of a minimal surface not discussed in class. Find a good picture of this surface, print it, and attach it to your homework. Explain this minimal surface, i.e., what minimization problem does it solve?

(2) (15 points) Find the extremals of the functional

$$J[y, z] = \int_0^{\pi/2} (y'^2 + z'^2 + 2yz) dx$$

subject to the boundary conditions

$$y(0) = 0 \quad y(\pi/2) = 1 \quad z(0) = 0 \quad z(\pi/2) = 1.$$

(3) (10 points) Consider the functional

$$J[w(x, y, z)] = \iiint_R f(x, y, z, w, w_x, w_y, w_z) dx dy dz$$

Derive the Euler-Lagrange equation for this functional. (Note: you will need to use Green's theorem as in the case for  $w(x, t)$  we did in class.)

(4) (10 points) The Dirichlet integral is

$$J[f(x, y)] = \iint f_x^2 + f_y^2 dx dy$$

Show that the extremal for this integral satisfies Laplace's equation. (This integral arises in many areas of physics and engineering, steady-state heat transfer, fluid flow, electrostatics, etc.)

(5) (50 points) On the course web page you will find a PDF document containing a list of articles. Pick one article that interests you, read it, and write a brief (1-2 page) abstract of the article. Your abstract should be focused on how the article relates to the calculus of variations. Your abstract should be typed. You should include a description of the problem the authors are trying to solve, the quantity they are minimizing, the functional they study, and a brief summary of their results. Again, you should focus on relating the material in the paper to what you have learned in this class.