

Homework 5
Math 352 - Spring 2003
Prof. John A. Pelesko

This assignment is to be handed in on May 19th. You are encouraged to work together, but the work you hand in must be your own.

1. Consider the equation governing the motion of a damped string

$$u_{tt} + \beta u_t = u_{xx}$$

Assume the string is held fixed at $x = 0$ and $x = 1$. Assume initially it satisfies

$$u(x, 0) = f(x)$$
$$u_t(x, 0) = g(x).$$

- (a) Solve using separation of variables.
- (b) If $g(x) = 0$, show that the solution may be written

$$u(x, t) = \frac{1}{2}(F(x - t) + F(x + t))$$

where F is the odd periodic extension of f .

- (c) If $f(x) = 0$, show that the solution may be written

$$u(x, t) = \frac{1}{2} \int_{x-t}^{x+t} G(z) dz$$

where G is the odd periodic extension of g .

- (d) Now, use your solution to model a plucked guitar string. Proceed as follows: Assume $g(x) = 0$ and that

$$f(x) = ax$$

for $0 \leq x \leq x_0$ and

$$f(x) = ax_0 \frac{1-x}{1-x_0}$$

for $x_0 \leq x \leq 1$. How does your solution depend on a and x_0 ? Discuss (from your solution) how where and how hard you pluck the string effects the sound.