**Homework 3**
**Math 352 - Spring 2003**
**Prof. John A. Pelesko**

This assignment is to be handed in on 4/9. You are encouraged to work together, but the work you hand in must be your own.

1. (20 points) Which of the following functions satisfy the 2-d Laplace equation?

   (a) $\phi = x^2 - y^2$

   (b) $\phi = x^3 + y^3$

   (c) $\phi = x$

   (d) $\phi = \frac{1}{2}\log(x^2 + y^2)$

2. (20 points) For each of the answers from question (1) that *did* satisfy the Laplace equation, assume that $\phi$ represents a potential for a potential flow. Compute the velocity vector field. That is, compute $\vec{v} = \nabla\phi$. Sketch this velocity field. You may use a computer to complete your sketch. (The bulk of the credit for this problem depends on your sketches.)

3. (30 points) Consider potential flow past a circular cylinder of radius 1. Assume that far away from the cylinder the flow is uniform and in the $x$-direction with velocity $V$.

   (a) Write down the pde and boundary conditions satisfied by the velocity potential $\phi$.

   (b) Verify that $\phi = U(x + \frac{x}{x^2+y^2})$ satisfies all the conditions of part (a).

   (c) From $\phi$ compute the velocity field $\vec{v}$ and sketch.

4. (30 points) Consider heat flow in a one dimensional rod of length $L$. Assume the ends of the rod are *insulated*. Assume that initially the temperature distribution in the rod is given by $T(x', 0) = T_0(1 + \cos(2\pi x'/L))$.

   (a) Write down the pde, boundary, and initial conditions satisfied by the temperature distribution, $T(x', t')$.

   (b) Scale your equations from part (a) to arrive at a dimensionless version of the problem.

   (c) Use the Laplace transform to solve your scaled problem from part (b).