Homework 2
Math 352 - Spring 2003
Prof. John A. Pelesko

This assignment is to be handed in on 3/12. You are encouraged to work together, but the work you hand in must be your own.

1. Compute the divergence, curl, and divergence of the curl for

   (a) $\vec{v} = (0, \sinh(xyz), 0)$

   (b) $\vec{v} = (x^2, y^2, z^2)$

   (c) $\vec{v} = \nabla\phi$ where $\phi = x\cos(x + y + z)$

2. Consider $\phi(x, y, z) = 2xz + e^y z^2$. Find the maximum and minimum rates of change of ϕ at the point $(2, 1, 1)$. Find the vectors that point in the direction of the maximum and minimum rates of change of ϕ.

3. Suppose $\phi(x, y, z)$ is a continuous function with continuous partial derivatives. Then, $\nabla\phi$ points in the direction normal to the surface $\phi = c$ at any point where $\nabla\phi$ is nonzero. Use this fact to find the normal vector to the surface $z = \sqrt{x^2 + y^2}$ at the point $(1, 1, \sqrt{2})$. Sketch this surface and the normal vector at this point.

4. Consider a solid body immersed in a fluid of constant density ρ. Archimedes's principle states that the net pressure force on the body is upward and equal to the weight of the fluid displaced by the body. Derive Archimedes principle. You may assume that the pressure distribution in the fluid as a function of depth z is $p(x, y, z) = -\rho g z$ where g is the gravitational acceleration and z is measured upwards from the surface of the fluid. Hint: Write the net force on the body as a surface integral and use the divergence theorem.

5. In the static case one of the four Maxwell equations reduces to

   $$\nabla \times \vec{E} = 0$$

   where $\vec{E}$ is called the *electric field*. In this static case $\vec{E}$ can be represented as a gradient of a potential, i.e., $\vec{E} = \nabla\phi$ for some scalar function ϕ. Show by direct calculation that the Maxwell equation above is automatically satisfied by such a representation of $\vec{E}$.