Exam # 1
Math 352 - Spring 2003
Prof. John A. Pelesko

This is the second exam of the semester. Please carefully and clearly show all of your work. No calculators, books, notes, or friends are allowed. Note that $\mathcal{L}$ denotes the Laplace transform. A table of transforms appears on the back of this paper. Good luck!

1. (40 points) Consider heat flow in a one-dimensional rod. Assume the problem has been scaled so that the dimensionless temperature satisfies the partial differential equation

$$u_t = u_{xx} \quad \text{on} \quad 0 < x < 1$$

and the initial condition

$$u(x, 0) = \cos(2\pi x).$$

(a) (10 points) If at $x = 0$ the temperature is held fixed at zero, and if at $x = 1$ the rod is insulated, state the proper boundary conditions for this problem.

(b) (30 points) Solve for $u(x, t)$ using the *Laplace transform method*. You do not need to *invert*, that is, you can simply solve for $\hat{u}(x, s)$.

2. (40 points) Consider the function $f(x) = x^2$ on the interval $[0, \pi]$.

(a) (5 points) Sketch the odd periodic extension of $f(x)$.

(b) (5 points) Sketch the even periodic extension of $f(x)$.

(c) (20 points) Compute the Fourier *sine* series of $f(x)$.

(d) (5 points) What does your sine series converge to at $x = \pi$? Why?

(e) (5 points) What would the Fourier cosine series of $f(x)$ converge to at $x = \pi$? Why?

3. (20 points) Suppose a vector field $\vec{v}$ satisfies

$$\nabla \times \vec{v} = 0$$
$$\nabla \cdot \vec{v} = \rho(x, y, z)$$

and we define a scalar potential $\vec{v} = \nabla\phi$.

(a) (10 points) Derive a scalar partial differential equation satisfied by ϕ.

(b) (10 points) If $\rho = 1$, which of the following satisfy your scalar equation from part (a)?

$$\phi = 1 \qquad \phi = \sin(x) - \cos(y)$$

$$\phi = x - y \qquad \phi = \frac{3}{2}x^2 - y^2$$