Exam # 1
Math 352 - Spring 2003
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This is the first exam of the semester. Please carefully and clearly show all of your work. No calculators, books, notes, or friends are allowed. Note that $\mathcal{L}$ denotes the Laplace transform. A table of transforms appears on the back of this paper. Good luck!

1. (10 points) State the divergence theorem.

2. (30 points) Use the definition of the Laplace transform to

   (a) Show that $\mathcal{L}\{e^{-at}f(t)\} = F(s+a)$.

   (b) Using part (a) show that $\mathcal{L}^{-1}\{F(s+b)\} = e^{-bt}f(t)$.

3. (30 points) Find the Laplace transform of $1 + H(t-2)e^{2t}$.

4. (30 points) Consider the initial value problem

$$x'' + 2x' = \delta(t-3)$$
$$x(0) = 1 \quad x'(0) = -1.$$

   (a) Show that $X(s) = \frac{e^{-3s}}{s(s+2)} + \frac{1}{s(s+2)} + \frac{1}{s+2}$.

   (b) Invert $X(s)$ to obtain $x(t)$.

   (c) Sketch $x(t)$.

LAPLACE TRANSFORM TABLE

$f(t) = \mathcal{L}^{-1}\{F(s)\}(t)$		$F(s) = \mathcal{L}\{f(t)\}(s) = \int_0^\infty e^{-st} f(t)\,dt$	
1		$\frac{1}{s}$,	$s > 0$
t^n ,	n an integer	$\frac{n!}{s^{n+1}}$,	$s > 0$
e^{at}		$\frac{1}{s-a}$,	$s > a$
$\sin bt$		$\frac{b}{s^2+b^2}$,	$s > 0$
$\cos bt$		$\frac{s}{s^2+b^2}$,	$s > 0$
$e^{at} f(t)$		$F(s-a)$	
$e^{at} t^n$	n an integer	$\frac{n!}{(s-a)^{n+1}}$,	$s > a$
$e^{at} \sin bt$		$\frac{b}{(s-a)^2+b^2}$,	$s > a$
$e^{at} \cos bt$		$\frac{(s-a)}{(s-a)^2+b^2}$,	$s > a$
$t \sin bt$		$\frac{2bs}{(s^2+b^2)^2}$	$s > 0$
$t \cos bt$		$\frac{s^2-b^2}{(s^2+b^2)^2}$ * ,	$s > 0$
$u_c(t) f(t),\ c \geq 0$		$e^{-cs}\mathcal{L}\{f(t+c)\}(s)$	
$u_c(t) f(t-c),\ c \geq 0$**		$e^{-cs}\mathcal{L}\{f(t)\}(s)$	
$y' = \dot{y} = \frac{dy}{dt}$		$sY(s) - y(0)$	
$y'' = \ddot{y} = \frac{d^2y}{dt^2}$		$s^2Y(s) - sy(0) - \dot{y}(0)$	

* **NB.** $\frac{b^2}{(s^2+b^2)^2} = \frac{\frac{1}{2}}{s^2+b^2} - \frac{\frac{1}{2}(s^2-b^2)}{(s^2+b^2)^2}$

** **Definition:** $u_c(t) = \begin{cases} 0 & t < c \\ 1 & t \geq c \end{cases}$ which is also written as $u(t-c)$ or $H(t-c)$