

Sample Exam 1
Math 341 - Fall 2002
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This is a practice test for exam #1 of the Fall 2002 semester. Be forewarned, this is a sample only, it is not meant to offer a complete review of the material you must master for the first exam. That is, while you should be able to solve all problems on this exam, your ability to solve problems should not be limited to those on this exam.

(1) (25 points) A 50 gallon tank initially contains 20 gallons of pure water. Salt-water solution containing 0.5lb of salt for each gallon begins to enter the tank at a rate of 4 gal/min. Simultaneously, a drain is opened at the bottom of the tank, allowing the salt-water to leave the tank at a rate of 2 gal/min. How many pounds of salt are in the tank at the precise moment that the tank is full of salt-water solution?

(2) (25 points) Solve the initial value problem

$$\frac{dx}{dt} = x \sin(t) + 2te^{-\cos(t)}, \quad x(0) = 1$$

(3) (25 points) Find all *equilibrium* solutions of

$$\frac{dx}{dt} = (x^2 - 1)(x - 2)$$

and determine the stability of each solution.

(4) (25 points) Solve the differential equation

$$\frac{dy}{dx} = y(y - 2).$$

Now, find the solution which also satisfies $y(0) = 3$. Suppose I had said find the solution which satisfies $y(0) = 0$? Would you be able to complete the problem? Why or why not?

(Bonus Extra Credit Problem) Prove that any separable equation is also exact. Recall that a separable equation may be written in the form

$$M(x) + N(y)y' = 0.$$