

Exam 1
Math 341 - Fall 2002
Prof. John A. Pelesko

This is exam #1 of the Fall 2002 semester. Show all your work and be sure to clearly indicate your final answer. Good luck!

(1) (15 points) Find all *equilibrium* solutions of

$$\frac{dy}{dt} = y - y^3$$

and determine the stability of each solution.

(2) (20 points) Solve the initial value problem

$$\frac{dy}{dt} = y - y^3, \quad y(0) = 1/2.$$

What is the limit of $y(t)$ as t tends to infinity?

(3) (25 points) A tank contains 50 liters of water. Salt water containing c grams per liter of salt enters the tank at the rate of 1.5 liters per minute. An open drain at the bottom of the tank allows the solution to leave at the rate of 1 liter per minute. If, after 20 minutes, you want the concentration of salt in the tank to be 20 grams per liter, what value should you choose for c ?

(4) (20 points) Solve

$$(2xy + e^y)dx + (x^2 + xe^y)dy = 0.$$

(5) (20 points) Solve

$$\sin(x) \frac{dy}{dx} + 2 \cos(x)y = \sin(x) \cos(x)$$

(Hint: Think log's.)

(Bonus Extra Credit) (Warning! Credit given only for 100% correct answers!) Suppose a hawk, P , at the point $(a, 0)$ spots a pigeon, Q , at the origin flying along the y -axis with speed v . The hawk flies toward the pigeon at speed w . Write down an ode describing the path of flight of the hawk.