**Sample Final Exam Problems**
**Math 341 - Fall 2002**
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These are practice problems for the final exam of the Fall 2002 semester. Be forewarned, this is a sample only, it is not meant to offer a complete review of the material you must master for the exam. That is, while you should be able to solve all problems on this exam, your ability to solve problems should not be limited to those on this exam.

(1)Define *linear transformation.*

(2) Let

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$$

(a) Which matrix represents $A^{-1}$?

$$\begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}, \qquad \begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix}$$

(b) Use $A^{-1}$ to solve $A\vec{x} = \vec{b}$ for

$$\vec{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

and for

$$\vec{b} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

(3) Let $f$ be the linear transformation that maps $\mathrm{R}^3$ to $\mathrm{R}^2$ in such a way that

$$f(\vec{e_1}) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad f(\vec{e_2}) = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \quad f(\vec{e_3}) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

find

$$f\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

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(4) Let $f$ be the linear transformation from problem (3) and $g : \mathrm{R}^2 \to \mathrm{R}^2$ be the linear transformation defined such that

$$g(\vec{e_1}) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad g(\vec{e_2}) = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

Find $f \circ g$ and apply to the vector

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

(5) Use Gaussian elimination to find all solutions of

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 1 \\ 2x_1 + x_2 - x_3 &= 2 \\ x_1 - 4x_2 - 2x_3 &= 1 \\ 5x_1 - 8x_2 - 2x_3 &= 5 \end{aligned}$$

(6) Find all *equilibrium* solutions of

$$y' = \sin(y)$$

and determine the linear stability of each solution.

(7) Solve

$$y' = y + x^2, \qquad y(0) = 1.$$

(8) Solve

$$xydx + (x^2 + 2y^2 + 2)dy = 0.$$

(9) A tank contains 100 gallons of fresh water. Salt water containing 2lbs per gallon of salt runs into the tank at the rate of 4 gallons per minute. The mixture runs out of the tank at the rate of 2 gallons per minute. Find the amount of salt present when the tank has 120 gallons of the mixture. Find the concentration of salt in the tank after 20 minutes have elapsed.

(10) Consider the differential equation

$$y' = x + y, \quad y(0) = 1$$

Using Euler's method, compute y(0.3). Take your step size to be 0.1.

(11) Find the general solution of

$$y'' - 2y' - 3y = 64xe^{-x}.$$

(12) A mass hung on a spring in a vacuum (i.e., with no damping) oscillates with a period of one second. When the mass-spring system is placed in air (i.e., with damping) its damped period is found to be two seconds. Find the differential equation for the damped motion.

(13) An object of 1 gram mass is hanging at rest on a spring that is stretched by 25 centimeters by the weight. A force of $0.01\sin(2t)$ Newtons is applied to the system and air damping provides a damping constant of 0.02162 kilograms per second. If the system is started from rest, solve for the motion of the mass. Find the steady-state motion. Plot.