

Sample Exam 3
Math 341 - Fall 2002
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This is a practice test for exam #3 of the Fall 2002 semester. Be forewarned, this is a sample only, it is not meant to offer a complete review of the material you must master for the exam. That is, while you should be able to solve all problems on this exam, your ability to solve problems should not be limited to those on this exam.

- (1)(25 points) A spring-mass system has a spring constant of 3N/m. A mass of 2kg is attached to the spring and the motion takes place in a viscous fluid that offers a resistance equal to the magnitude of the instantaneous velocity. The system is driven by an external force of $3 \cos(\omega t)$.
- (a) Write the equation of motion for this system.
 - (b) Assume the system starts from rest with no initial displacement. What are the appropriate initial conditions for this case? Solve for the motion of the system.
 - (c) What is the steady-state response of the system?
 - (d) At what forcing frequency ω will the amplitude of the steady-state response be greatest?
 - (e) Plot a frequency response curve for the steady-state response of this system.
- (2) (20 points) Consider a linear transformation $f : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ that rotates vectors by ninety degrees and stretches their length by a factor of 2. Write the matrix corresponding to this linear transformation.
- (3) (20 points) Use Gaussian elimination to find all solutions of

$$\begin{aligned}x_1 - 3x_2 + x_3 &= 1 \\2x_1 + x_2 - x_3 &= 2 \\x_1 + 4x_2 - 2x_3 &= 1 \\5x_1 - 8x_2 + 2x_3 &= 5\end{aligned}$$

- (4) (20 points) Consider a linear system with augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ -1 & 4 & 3 & 2 \\ 2 & -2 & a & 3 \end{array} \right]$$

For what values of a will the system have a unique solution?

- (5) (15 points) Suppose $f : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ and $g : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ are linear transformations. Prove that $f + g$ is a linear transformation.