

**Final Exam**  
**Math 341 - Fall 2002**  
**Prof. John A. Pelesko**

Feel free to use the scrap paper provided, but make sure you show your final answer in the space provided. If you do not have enough space on the exam paper to show all your work, show it on attached clearly labelled scrap paper. No calculators, books, notes, or friends are allowed. Good luck!

Name:

Problem	Score	Value
1		10
2a		10
2b		10
3a		10
3b		5
3c		5
4a		20
4b		5
5a		5
5b		10
5c		5
5d		5

(1) (10 points) Define *linear transformation*.

(2a) (10 points) Consider the  $3 \times 3$  matrices

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} a & 1/2 & 1/4 \\ 1/2 & b & 1/2 \\ 1/4 & 1/2 & c \end{pmatrix}$$

Find  $a$ ,  $b$ , and  $c$  so that  $B = A^{-1}$ .

(2b) (10 points) Use the result of part (a) to solve  $A\vec{x} = \vec{b}$  for

$$\vec{b} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$$

(3a) (10 points) Find the general solution of

$$x \frac{dy}{dx} - 2y = x^3 \cos(4x)$$

(3b) (5 points) Find the solution of the equation from (3a) that satisfies  $y(2\pi) = 0$ .

(3c) (5 points) Can you find the solution of the equation of (3a) that satisfies  $y(0) = 1$ ? Explain why or why not.

(4a) (20 points) A tank initially holds 80 gallons of a brine solution containing  $1/8$  lb of salt per gallon. At time  $t = 0$  another brine solution containing 1 lb of salt per gallon is poured into the tank at the rate of 4 gallons per minute, while the mixture leaves the tank at the rate of 8 gallons per minute. Find the amount of salt in the tank at any time,  $t$ .

(4b) (5 points) Determine when the tank will contain the most salt.

(5) **In this problem take the gravitational constant  $g = 10\text{m/s}^2$ .** A 9kg mass is suspended from the end of a spring. The mass stretches the spring by 90 meters. The mass-spring system is placed in a viscous fluid that causes a damping force proportional to the velocity of the mass. The damping constant is 10 kg/s. The system is forced sinusoidally by an external force of  $2\sin(t/3)$  N.

(a) (5 points) Write the equation of motion of the mass-spring system.

(b) (10 points) Find the general solution of the equation from part (a).

(c) (5 points) If the mass-spring system is initially displaced 4m and started with a velocity of 1 m/s, solve for the motion of the system.

(d) (5 points) Find the steady-state behavior of the system from part (c) and plot.

(Bonus Question) Credit will only be given for 100% correct answers! The English alphabet is symbolized by  $1, 2, \dots, 26$  and a  $2 \times 4$  message matrix is encoded by multiplying it by the matrix

$$\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}.$$

Decode the message received, namely

$$\begin{pmatrix} 53 & 45 & 51 & 26 \\ 129 & 105 & 120 & 63 \end{pmatrix}.$$