Homework 5
Math 302 - Fall 2006
Prof. John A. Pelesko
Due October 11, 2006

This homework assignment is different than others you have had this semester. Most of the questions for this assignment require written essay style answers. I expect your answers to be written clearly, to be intelligible, to be complete, and to be precise.

(1) (10 points) What is a differential equation? Write a precise definition.

(2) (10 points) Give three examples of physical laws that can be expressed as differential equations. Write these equations down and explain the meaning of each term in the equation.

(3) (10 points) What is the most general first order autonomous differential equation? Write it down. Explain how the solutions to this equation can be analyzed in terms of critical points and their stability.

(4) (10 points) What is the most general linear first order differential equation? Explain how to solve this equation in the general case.

(5) (10 points) What is the most general second order differential equation? What is the most general second order linear differential equation?

(6) (10 points) Explain Euler's method for numerically solving differential equations.

(7) (20 points) A steel ball weighing 128 pounds is suspended from a spring, whereupon the spring stretches 2 feet from its natural length. The ball is started in motion with no initial velocity by displacing it 6 inches above the equilibrium position. Assuming no damping in the system, find the position of the ball as a function of time. Plot. Find the position at time $t = \pi/12$ seconds.

(8) (20 points) Consider a solid cylinder placed vertically in a tub of water. Assume the density of the cylinder, ρ, is less than that of water. Write an equation to describe the vertical motion of the cylinder. You may assume there is no damping in the system. (Hint: You need Archimedes principle and Newton's 2nd Law.)